Applications of Integrals

February 7, 2006

Average of a function

- If $f : [a,b] \to \mathbb{R}$ is integrable, then
$$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{\int_a^b f(x)\,dx}{\text{length of interval } [a,b]}.$$

- If $f : D \subseteq \mathbb{R}^2 \to \mathbb{R}$ is integrable, then
$$\frac{\iint_D f(x,y)\,dA}{\iint_D dA} = \frac{\iint_D f\,dA}{\text{area of } D}$$

- If $f : W \subseteq \mathbb{R}^3 \to \mathbb{R}$ is integrable, then
$$\frac{\iiint_W f(x,y,z)\,dV}{\iiint_W dV} = \frac{\iiint_W f\,dV}{\text{volume of } W}$$

Total Mass

If W is a solid with density $\delta(x, y, z)$ then its **mass** is

$$\iiint_W \delta(x, y, z)\, dV$$

Center of Mass in $\mathbb{R}^2$

For a lamina D with density function $\delta(x, y)$ the **center of mass** is

$$\begin{aligned}
\bar{x} &= \frac{\text{total moment with respect to } y\text{-axis}}{\text{total mass}} \\
&= \frac{\iint_D x\delta(x, y)\, dA}{\iint_D \delta(x, y)\, dA} \\
\bar{y} &= \frac{\text{total moment with respect to } x\text{-axis}}{\text{total mass}} \\
&= \frac{\iint_D y\delta(x, y)\, dA}{\iint_D \delta(x, y)\, dA}
\end{aligned}$$

Center of Mass in $\mathbb{R}^3$

W a solid with density $\delta(x,y,z)$.

$$\bar{x} = \frac{\text{total moment with respect to } yz\text{-plane}}{\text{total mass}}$$

$$= \frac{\iiint_W x\delta(x,y)\,dV}{\iiint_W \delta(x,y,z)\,dV}$$

$$\bar{y} = \frac{\text{total moment with respect to } xz\text{-plane}}{\text{total mass}}$$

$$= \frac{\iiint_W y\delta(x,y)\,dV}{\iiint_W \delta(x,y,z)\,dV}$$

$$\bar{z} = \frac{\text{total moment with respect to } xy\text{-plane}}{\text{total mass}}$$

$$= \frac{\iiint_W z\delta(x,y,z)\,dV}{\iiint_W \delta(x,y,z)\,dV}$$