Green's Theorem

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Green's Theorem

D is closed bounded region and $C = \partial D$ its boundary. Let $\mathbf{F}(x, y) = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$ a vector field. Then

$$\boxed{\oint_C M\,dx + N\,dy = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)\,dxdy}$$

WARNING: Here C must be oriented so that D is on the left as one traverses C.

Vector Formulation of Green's Theorem

$$\mathbf{F}(x, y) = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$$

$$\boxed{\oint_{\partial D} \mathbf{F} \cdot d\mathbf{s} = \iint_D (\nabla \times \mathbf{F}) \cdot \mathbf{k}\, dA}$$

Recall: $\nabla \times \mathbf{F}$ is the curl of $\mathbf{F}$.

Divergence Theorem in the plane

D is a closed bounded region and $\mathbf{n}$ is the outward unit normal vector to D and $\mathbf{F} = M(x,y)\mathbf{i} + N(x,y)\mathbf{j}$, then

$$\boxed{\oint_{\partial D} \mathbf{F} \cdot \mathbf{n}\, ds = \iint_D \nabla \cdot \mathbf{F}\, dA.}$$

Recall: $\nabla \cdot \mathbf{F}$ is the divergence.