Stokes' and Gauss' Theorems

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Stokes' Theorem

Let S be a bounded, piecewise smooth oriented surface in $\mathbb{R}^3$. Assume ∂S consists of simple closed curves oriented consistently with S. Let $\mathbf{F}$ be a C^1 vector field. Then

$$\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S} = \oint_{\partial S} \mathbf{F} \cdot d\mathbf{s}$$

Gauss' Theorem

Let W be a bounded solid region in $\mathbb{R}^3$ whose boundary ∂W consists of smooth, closed orientable surfaces, each oriented so that $\mathbf{n}$ (unit normal) points **away** from W. Let $\mathbf{F}$ be a class C^1 vector field. Then

$$\oiint_{\partial W} \mathbf{F} \cdot d\mathbf{S} = \iiint_W \nabla \cdot \mathbf{F} \, dV$$

Surface Independence

Let $\mathbf{G}$ be a vector field defined on a region R in $\mathbb{R}^3$. If either

(a) $\mathbf{G} = \text{curl } \mathbf{F}$ for some $\mathbf{F}$ or

(b) div $\mathbf{G} = 0$ and R is all of $\mathbb{R}^3$, then

$$\iint_{S_1} \mathbf{G} \cdot d\mathbf{S} = \iint_{S_2} \mathbf{G} \cdot d\mathbf{S}$$

whenever S_1 and S_2 are two oriented surfaces in R such that $\partial S_1 = \partial S_2$.

Path Independence and FT of line integrals

A vector field $\mathbf{F}$ on a region in $\mathbb{R}^n$ is the gradient of a some function if and only if, for any two paths C_1 and C_2 with the same endpoints

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{s} = \int_{C_2} \mathbf{F} \cdot d\mathbf{s}$$