Matrices and Coordinates

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Matrix Multiplication

If A is an $m \times n$ matrix and B is an $n \times p$ matrix then the **product** AB is the matrix where the ij-th entry is obtained by taking the dot product of the i-th row of A with the j-th column of B.

NOTE: In order to define the product of A and B we require that the number of columns of A be equal to the number or rows of B. Otherwise, the product is undefined.

Linear Mappings

A **linear mapping** $F : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is defined as

$$F(\mathbf{x}) = A\mathbf{x}$$

for every $\mathbf{x}$ in $\mathbb{R}^n$ where A is an $m \times n$ constant matrix.

Linear mappings are VERY important in all applications in science and engineering and in mathematics.

Properties of linear transformations

If $F : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a linear mapping then for any $\mathbf{x}, y$ in $\mathbb{R}^n$ and any scalar c we have

$$F(\mathbf{x}+\mathbf{y}) = F(\mathbf{x}) + F(\mathbf{y})$$

$$F(c\,\mathbf{x}) = c\,F(\mathbf{x}).$$

Coordinate Systems

The **coordinates** of a point are the components of a tuple of numbers used to represent the location of the point in the plane or space.

For 2-dimensions:

- Choose an "origin" - (0,0).
- **Cartesian or rectangular coordinates**

$$(x, y)$$

x-horizontal and y-vertical direction

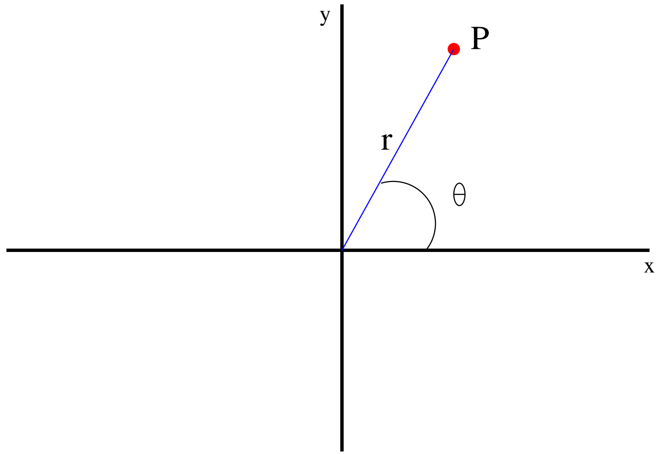
Polar coordinates

$(\mathbf{r},\theta)$: r -distance from origin and
θ-angle from x-axis, $0 \leq \theta < 2\pi$.
If we want to describe every point uniquely we require that $r \geq 0$ and $0 \leq \theta < 2\pi$.

NOTE: In polar coordinates you think that every point except the origin is on a circle of radius r.

Polar coordinates are useful in doing computations with curves that have symmetry around the origin.

Relation between polar and cartesian coordinates



Polar to Cartesian:

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Cartesian to Polar:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(\frac{y}{x})$$

Cylindrical Coordinates

These are for 3D: (r, θ, z) and we usually think that every point in space not on the z-axis is on a cylinder.

They are good for studying objects possessing an axis of symmetry.

Cartesian to Cylindrical	**Cylindrical to Cartesian**
$x = r\cos\theta$	$r = \sqrt{x^2 + y^2}$
$y = r\sin\theta$	$\theta = \tan^{-1}(\frac{y}{x})$
$z = z$	$z = z$

Spherical Coordinates

- These coordinates are also to describe a point in 3D: (ρ, ϕ, θ). They are useful to study objects that have a center of symmetry.

- Here we think as every point except (0,0,0) lies on a sphere.

- ρ - distance from the origin.
  ϕ - longitude and takes values $0 \leq \phi \leq \pi$.
  θ - latitude and takes values $0 \leq \theta < 2\pi$.

Relation between cartesian and spherical

Spherical to cartesian:

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

Cartesian to spherical:

$\rho = \sqrt{x^2 + y^2 + z^2}$

$\phi = \tan^{-1}(\sqrt{x^2 + y^2}/z)$

$\theta = \tan^{-1}(y/x)$.

Relation between cylindrical and spherical

Spherical to cylindrical:

$r = \rho \sin\phi$

$z = \rho \cos\phi$

$\theta = \theta.$

Cylindrical to spherical:

$\rho = \sqrt{r^2 + z^2}$

$\phi = \tan^{-1}(r/z)$

$\theta = \theta.$