WRITTEN HOMEWORK #7, DUE 2/24/2012 AT 4PM

You may turn this assignment at the homework boxes on the bottom floor of Kemeny or at the beginning of class on Friday. Please staple your assignment before turning it in. Remember that you need to provide correct and reasonably complete details to receive full credit. The problems are taken from the 7th edition of Stewart's *Calculus*, although occasionally a problem will be modified to be slightly different from its textbook counterpart.

(1) (Chapter 16.5, Problem #12) Let f be a scalar function and $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ a vector field. State whether each expression below is meaningful. If not, explain why, and if so, state whether it is a scalar function or a vector field, and give a brief explanation why.
   (a) curl f,
   (b) grad f,
   (c) div $\mathbf{F}$,
   (d) curl(grad f),
   (e) grad $\mathbf{F}$,
   (f) grad(div $\mathbf{F}$),
   (g) div(grad f),
   (h) grad(div f),
   (i) curl(curl $\mathbf{F}$),
   (j) div(div $\mathbf{F}$),
   (k) grad(f) $\times$ (div $\mathbf{F}$),
   (l) div(curl(grad f))).

(2) (Chapter 16.5, Problem #20) Is there a smooth vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that $\nabla \times \mathbf{G} = \langle xyz, -y^2z, yz^2 \rangle$? Explain.

(3) (Chapter 16.6, Problem #24) Find a parametric representation for the surface which is the part of the sphere $x^2 + y^2 + z^2 = 16$ which lies between the planes $z = -2$ and $z = 2$.

(4) (Chapter 16.6, Problem #36) Let $\mathbf{r}(u, v) = \langle \sin u, \cos u \sin v, \sin v \rangle$. Find an equation for the tangent plane to this surface at $u = \pi/6, v = \pi/6$. (For fun, you might want to try to sketch this surface by hand, or at least look at a picture of it on a computer. Do not turn your drawing in, however.)

(5) (Chapter 16.6, Problem #64a) Find a parametric representation for the torus obtained by rotating about the z-axis the circle in the xz-plane with center $(b, 0, 0)$ and radius $a < b$. (See the textbook for a picture and a relevant hint.)