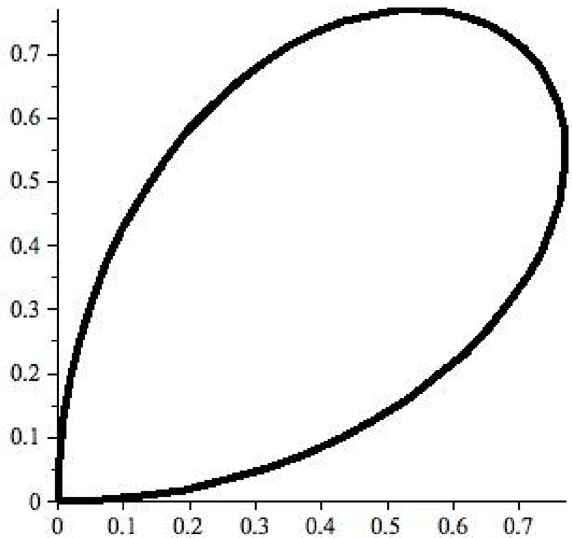
1. (15) Evaluate

$$\int_0^2 \int_y^2 e^{x^2}\, dx\, dy.$$

(Hint: You can't write down an antiderivative for e^{x^2} with respect to x.)

2. (10) The graph of $r = \sin 2\theta, 0 \le \theta \le \pi/2$, is sketched below. Find the area of the region enclosed by this graph.



3. (15) Let T be the transformation from uv coordinates to xy coordinates given by

$$x = u, y = \frac{v}{u}.$$

Let R be the rectangle $1 \leq u \leq 3, 1 \leq v \leq 2$.

(a) [5] Sketch $T(R)$. Label all the boundary curves of $T(R)$.

(b) [5] What is the Jacobian of T?

(c) [5] What is the area of $T(R)$?

4. (10) Let C be the curve as sketched below. It consists of a line segment from $(1,1)$ to $(4,4)$, a semicircular arc from $(4,4)$ to $(6,4)$, and then a line segment from $(6,4)$ to $(8,4)$. Let $\mathbf{F} = \langle e^{x-2y}, -2e^{x-2y} \rangle$. Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

5. (20) Let $\mathbf{F} = \langle P, Q \rangle$ be defined by

$$\mathbf{F} = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle .$$

(This is the same $\mathbf{F}$ that you have seen several times in class and on previous homework assignments.) Let C be the strangely shaped curve sketched below.

(a) [3] Check that $Q_x = P_y$.

(b) [5] Let C_a be a circle of radius a, centered at the origin, with counterclockwise orientation. Evaluate

$$\int_{C_a} \mathbf{F} \cdot d\mathbf{r}.$$

(c) [2] Briefly explain why you cannot apply Green's Theorem to C and its interior directly.

(d) [5] Consider C_1, C_2 as sketched below. The small hemispherical bump is part of the circle C_a, for small a. The line segments lie on the x axis, while the top of C_1 is the part of C above the x axis, and the bottom of C_2 is the part of C below the x axis. Evaluate

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} \text{ and } \int_{C_2} \mathbf{F} \cdot d\mathbf{r}.$$

(e) [3] Suppose we choose a so that C_a is completely contained in the interior of C. Briefly explain why

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} + \int_{C_2} \mathbf{F} \cdot d\mathbf{r} = \int_{C} \mathbf{F} \cdot d\mathbf{r} - \int_{C_a} \mathbf{F} \cdot d\mathbf{r}.$$

(f) [2] What is the value of

$$\int_{C} \mathbf{F} \cdot d\mathbf{r}?$$

6. (15) Let S be the graph of $f(x, y) = xy^2$, with orientation pointing upward, over the domain D given by $0 \leq x \leq 1, 0 \leq y \leq 2$, and let $\mathbf{F} = \langle x, xy, z \rangle$. Evaluate the surface integral of $\mathbf{F}$ across S.

7. (20) Let S be the solid triangle with vertices $(1,0,0),(0,1,0),(0,0,2)$.

(a)[5] Find an equation for the plane which contains S.

(b)[15] Evaluate

$$\iint\limits_S z\,dS.$$

8. (15) Let S be the cylinder $x^2 + y^2 = 9, 0 \leq z \leq 4$, with orientation pointing away from the z-axis (ie, pointing outward). Notice that S has no top or bottom. Let $\mathbf{F} = \langle e^{x^2+y^2}x, e^{x^2+y^2}y, xy^2z \rangle$.

(a)[5] Find an expression for the unit normal vector $\mathbf{n}$ to S which defines the given orientation on S at a point (x, y, z) on S, as a function of x, y, z.

(b)[5] For (x, y, z) on S, what is $\mathbf{F} \cdot \mathbf{n}$, as a function of x, y, z? Simplify your answer as much as possible.

(c)[5] Evaluate

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS.$$

(Hint: The surface area of a cylinder with no top or bottom of height h and radius r is $2\pi r h$.)

9. (15) Let E be the solid $(x^2+y^2)-1 \le z \le 1-(x^2+y^2), 0 \le x^2+y^2 \le 1$, and let S be the surface of this solid, with outward pointing orientation. Let $\mathbf{F} = \langle 2x\sin z, y^2, xz+e^y+2\cos z\rangle$. Evaluate

$$\iint\limits_S \mathbf{F}\cdot\mathbf{n}\, dS.$$

(Hint: You don't need to parameterize S to solve this problem.)

10. (25) Let S be the cone $z = 2 - \sqrt{x^2 + y^2}, 0 \leq z \leq 2$, with upwards pointing orientation. Let $\mathbf{F} = \langle -2y + x, x + y, \log(z + 1) \rangle$. Notice that $\mathbf{F}$ is only defined for $z > -1$.

(a)[5] Calculate $\nabla \times \mathbf{F}$.

(b)[10] Evaluate

$$\iint\limits_{S} (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS$$

directly. (That is, don't use Stokes' Theorem.)

(c)[10] Stokes' Theorem says that

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS = \int_C \mathbf{F} \cdot d\mathbf{r},$$

where C is the boundary curve of S with appropriate orientation. Verify Stokes' Theorem by directly calculating

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

and checking that your answer matches with that you found in part (b).

11. (15) For each of the following three vector fields, determine whether they are conservative on the given domain D or not, and explain your answer.

(a) [5] $\mathbf{F} = \langle \log y + \log z, \frac{x}{y}, \frac{x}{z} \rangle$, D is the set of points with $x, y, z > 0$.

(b) [5] $\mathbf{F} = \langle -2y + x, x + y, \log(z + 1) \rangle$ (this is identical to the vector field from the previous problem), D is the set of points with $z > -1$.

(c) [5]

$$\mathbf{F} = \left\langle \frac{-y}{x^2+y^2} + e^{x^2}, \frac{x}{x^2+y^2} + 3y \right\rangle,$$

D is $\mathbb{R}^2$ with the point $(0,0)$ removed.

Use this page for scratch work.

Use this page for scratch work.

NAME : ______________________________
SECTION : ______________________________

Math 13

March 12 2010, 11:30am - 2:30pm
Final Exam

INSTRUCTIONS: This is a closed book exam. You are not to provide or receive help from any outside source during the exam, but you may ask the instructor for clarification on questions. The exam is three hours long.

- Wait for signal to begin.
- ***Print*** your name in the space provided.
- Calculators, computers, or other computing devices are not allowed.
- In consideration of other students, please turn off cell phones or other electronic devices which may be disruptive.
- You must show your work and justify your solutions to receive full credit on all questions. Work that is illegible may not be graded; work that is scratched out will not be graded, even if it is correct.
- After the final exams are graded, you will not be able to take your final (this is Dartmouth policy), but you may visit the instructor's office to look at your final exam. You will also be able to find out your final grade for the class at that time. The final exams should be graded by Monday.
- Two trigonometric identities that might be helpful: $\cos^2 x = \dfrac{1+\cos 2x}{2}, \sin^2 x = \dfrac{1-\cos 2x}{2}$.

Problem	Points	Score
1	15	
2	10	
3	15	
4	10	
5	20	
6	15	
7	20	
8	15	
9	15	
10	25	
11	15	
Total	175	