1. (15) Let E be the solid bounded by $x \geq 0, y \geq 0, 0 \leq z \leq 1 - x^2 - y$.

(a) [5] Sketch the projection of E onto the xy, xz, and yz planes. You do not need to explain why your answer is correct, but label all intercepts and identify the curves on the boundary of the regions you sketch by writing down the functions they are given by.

(b) [5] Write the triple integral $\iiint\limits_{E} x \, dV$ as an iterated integral in each of the orders of integration $dz \, dy \, dx$, $dy \, dz \, dx$, $dx \, dy \, dz$.

(c) Evaluate the triple integral $\iiint\limits_E x\,dV$ using any of the orders you found in part (b).

2. (15) Consider the cone $0 \leq z \leq 2 - \sqrt{x^2 + y^2}$. Let E be the part of this cone with $0 \leq z \leq 1$ (that is, we cut away the top part of the cone). Below is a figure which indicates the shape of this solid: the base and top are both circles.

(a) [5] Suppose we want to evaluate $\iiint\limits_E f(x, y, z) \, dV$ using cylindrical coordinates. Write down a sum of two iterated integrals, in cylindrical coordinates, with order of integration $dz \, dr \, d\theta$, one of which has the bounds $0 \leq r \leq 1$ on r, and the other with $1 \leq r \leq 2$, which is equal to this triple integral.

(b) [5] With $f(x, y, z) = 1$, evaluate $\iiint\limits_{E} dV$ to find the volume of E.

(c) [5] The volume of a cone is given by the formula $V = \dfrac{bh}{3}$, where b is the area of the circular base and h the height of the cone with respect to this base. Use this formula from geometry (no integration allowed!) to compute the volume of E and check that your answer is the same as that in part (b).

3. (12) Suppose a change of variables T from the uv plane to the xy plane is given by the formulas

$$
\begin{aligned}
x &= u + 3v \\
y &= 2u + 5v
\end{aligned}
$$

(a) [4] Find the Jacobian of this change of variables.

(b) [4] Let S be the rectangle in the uv plane given by inequalities $0 \le u \le 2, 0 \le v \le 1$. You may assume that the image of a rectangle under the transformation T is a parallelogram. Find the vertices of this parallelogram.

(c) [4] What is the area of the parallelogram you found in (b)? Remember to explain how you found your answer.

4. (10) Let C be the semicircle $x^2 + y^2 = 4, y \geq 0$. Evaluate the line integral

$$\int_C xy + x + y \, ds.$$

5. (12) Let $\mathbf{F} = \langle y^2, x \rangle$.

(a) [5] Let C be the line segment starting at $(0,0)$ and ending at $(4,2)$ – that is, the orientation of C is given by starting at $(0,0)$ and ending at $(4,2)$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(b) [5] Let C now be given by the piece of the graph of $y = \sqrt{x}$ starting at $(0,0)$ and ending at $(4,2)$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(c) [2] Based on your answers to parts (a) and (b), is $\mathbf{F}$ conservative? Briefly explain why.

6. (10) Let $\mathbf{F}(x,y) = \langle y\cos x^2 + y^2 + 2xy, x^2 + 2xy + 2\rangle$. Let C be the closed curve given by the triangle starting at $(0,0)$, then going to $(2,2)$, then $(2,0)$, and then back to $(0,0)$. Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

7. (12) For each of the following four vector fields and corresponding domains D they are defined on, determine whether they are conservative on D or not, and explain why.

(a) $\mathbf{F}(x, y) = \langle 3xy, x^2 \rangle, D = \mathbb{R}^2$.

(b) $\mathbf{F}(x, y, z) = \langle x, y, z \rangle, D = \mathbb{R}^3$.

(c) $\mathbf{F}(x, y) = \langle e^{x^2} + y, e^{y^2} \rangle$, D is the annulus $1 \leq x^2 + y^2 \leq 4$.

(d) $\mathbf{F}(x, y) = \langle \dfrac{-y}{x^2 + y^2}, \dfrac{x}{x^2 + y^2} \rangle$, D is given by the inequalities $x > 0, -x < y < x$.
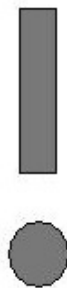
8. (6) For each of the three shaded regions below, indicate **to the right of the figure** whether they are **A**: connected but not simply connected, **B**: simply connected, or **C**: neither connected nor simply connected. You do not need to explain your answers. (Remember, write your answer to the right of the corresponding figure!)
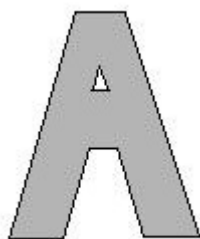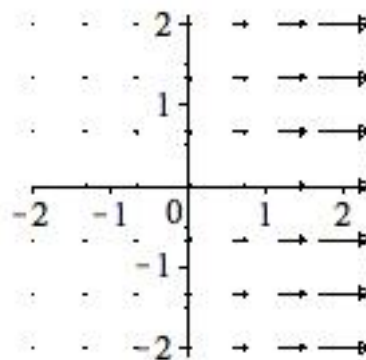

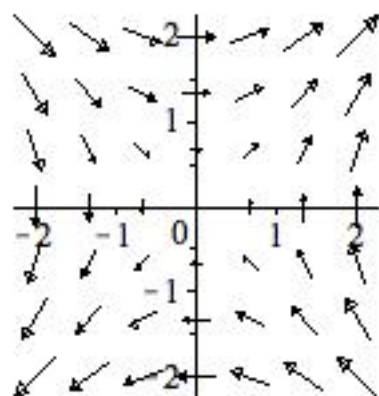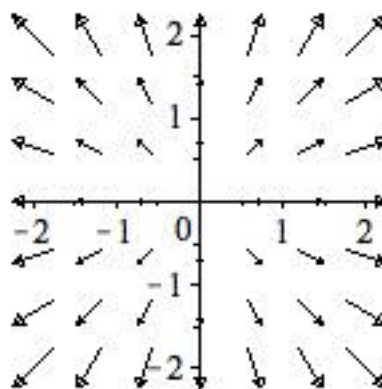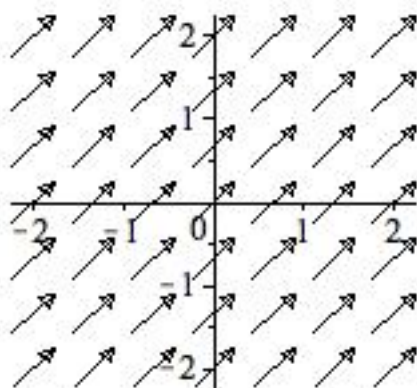(A bubble letter A)
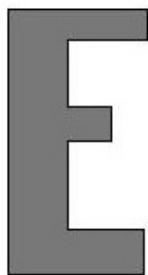
(A bubble letter E)

(A bubble exclamation point)

9. (8) Match each function with the plot of its gradient vector field. Write the letter attached to a function **underneath** its corresponding plot. You do not need to explain your answers. (Remember, write your answer underneath the corresponding plot!)

A : $f(x, y) = x + y$
B : $f(x, y) = x^2 + y^2$
C : $f(x, y) = xy$
D : $f(x, y) = e^x$

Use this page for scratch work.

Use this page for scratch work.

NAME : _____

SECTION : _____

Math 13

February 18 2010, 5pm - 7pm
Midterm #2

INSTRUCTIONS: This is a closed book exam. You are not to provide or receive help from any outside source during the exam. You may ask the instructor for clarification on questions. The exam is two hours long.

- Wait for signal to begin.

- **_Print_** your name in the space provided.

- Calculators, computers, or other computing devices are not allowed.

- In consideration of other students, please turn off cell phones or other electronic devices which may be disruptive.

- You must show your work and justify your solutions to receive full credit on all questions except #8 and #9. Work that is illegible may not be graded; work that is scratched out will not be graded, even if it is correct.

Problem	Points	Score
1	15	
2	15	
3	12	
4	10	
5	12	
6	10	
7	12	
8	6	
9	8	
Total	100	