Math 13, Homework #3

Due January 27, 2016

1. (12.4.42)

Find the area of the parallelogram determined by the vectors $< a, 0, 0 >$ and $< 0, b, c >$.

2. (15.6.22)

Let $G(u, v) = (u - uv, uv)$.
(a) Show that the image under G of the horizontal line $v = c$ is $y = \frac{c}{1-c}x$ if $c \neq 1$, and is the y-axis if $c = 1$.

(b) Determine the images under G of vertical lines in the uv-plane.

(c) Compute the Jacobian of G.

(d) Observe that by the formula for the area of a triangle, the region $\mathcal{D}$ bounded by the inequalities $x \geq 0, y \geq 0, x + y \geq a$, and $x + y \leq b$ has area $\frac{1}{2}(b^2 - a^2)$. Compute the area of $\mathcal{D}$ again, using the Change of Variables Formula applied to G.

(e) Calculate $\int\int_{\mathcal{D}} xy\, dx\, dy$.

3. (15.6.24)

Find a linear map T that maps $[0, 1] \times [0, 1]$ to the parallelogram $\mathcal{P}$ in the xy-plane with vertices $(0, 0), (2, 2), (1, 4), (3, 6)$. Then calculate the double integral of e^{2x-y} over $\mathcal{P}$ via change of variables.

4. (15.6.40)

Sketch the domain

$$\mathcal{D} = \{(x, y) : 1 \leq x + y \leq 4, -4 \leq y - 2x \leq 1\}.$$

(a) Let F be the map $u = x + y, v = y - 2x$ from the xy-plane to the uv-plane, and let G be its inverse.

(b) Compute $\int\int_{\mathcal{D}} e^{x+y}\, dx\, dy$ using the Change of Variables formula with the map G. *Hint:* It is not necessary to solve for G explicitly.