HOMEWORK 8 (DUE WEDNESDAY MARCH 2)

(1) (§16.4 Problem 40) Let $\mathcal{S}$ be the portion of the sphere $x^2 + y^2 + z^2 = 9$ where $1 \leq x^2 + y^2 \leq 4$ and $z \geq 0$ (see Figure 21 on p. 943). Find a parametrization of $\mathcal{S}$ in polar coordinates and use it to compute:

  (a) The area of $\mathcal{S}$.

  (b) $\displaystyle\iint_{\mathcal{S}} z^{-1}\, dS$.

(2) (§16.5 Problem 16) Let $\boldsymbol{F} = \langle 0, 0, z^2 \rangle$ and $\mathcal{S}$ be parametrized by $G(u, v) = (u \cos v, u, \sin v, v)$ for $0 \leq u \leq 1$, $0 \leq v \leq 2\pi$ wtih upward-pointing normal. Calculate

$$\iint_{\mathcal{S}} \boldsymbol{F} \cdot d\boldsymbol{S}.$$

(3) (§16.5 Problem 20) Show that the flux of $\boldsymbol{F} = \frac{\boldsymbol{e}_r}{r^2}$ through a sphere centered at the origin does not depend on the radius of the sphere.

(4) (§16.5 adapted from Problems 37 and 38) The surface $\mathcal{S}$ with parametrization

$$G(u, v) = \left(\left(1 + v \cos \tfrac{u}{2}\right) \cos u, \left(1 + v \cos \tfrac{u}{2}\right) \sin u, v \sin \tfrac{u}{2}\right)$$

is a Möbius strip.

  (a) The intersection of $\mathcal{S}$ with the xy-plane is the unit circle $(\cos u, \sin u, 0)$. Verify that the normal vector along this circle is

$$\boldsymbol{N}(u, 0) = \left\langle \cos u \sin \tfrac{u}{2}, \sin u \sin \tfrac{u}{2}, -\cos \tfrac{u}{2} \right\rangle.$$

  (b) As u varies from 0 to 2π, the point $G(u, 0)$ moves once around the unit circle, beginnning and ending at $G(0, 0) = G(2\pi, 0) = (1, 0, 0)$. Verify that $\boldsymbol{N}(u, 0)$ is a unit vector that varies continuously but that $\boldsymbol{N}(2\pi, 0) = -\boldsymbol{N}(0, 0)$ and explain why this shows that $\mathcal{S}$ is not orientable.

(5) (§17.3 Problem 8) Let $\boldsymbol{F} = \langle x^2 z, yx, xyz \rangle$ and $\mathcal{S}$ the boundary of the tetrahedron given by $x + y + z = 1,\ x \geq 0,\ y \geq 0,\ z \geq 0$. Use the divergence theoremo to calculate

$$\iint_{\mathcal{S}} \boldsymbol{F} \cdot d\boldsymbol{S}.$$

(6) (§17.3 Problem 23) Let $\mathcal{W}$ be the region bounded by the cylinder $x^2+y^2 = 4$, the plane $z = x + 1$, and the xy-plane (see Figure 19 on p. 992). Use the divergence theorem to compute the flux of $\boldsymbol{F} = \langle z, x, y + z^2 \rangle$ through the boundary of $\mathcal{W}$.