HOMEWORK 1

DUE: 13 JANUARY 2016

Please write solutions neatly using only one side of each page. Be sure that your solutions are readable and complete - a solution which consists only of equations, without explanation of what the equations mean and why they are true, will not receive full credit. When writing your solutions, be very mindful of the honor code section of the course website.

1. Problem # 1 (#41 in Section 15.1)

Evaluate the following integral

$$\int\int_{\mathcal{R}} e^x \sin(y) dA,$$

where $\mathcal{R} = [0, 2] \times [0, 4]$.

2. Problem # 2 (#50 in Section 15.1)

Prove the following extension of the Fundamental Theorem of Calculus to two variables: If $\frac{\partial^2 F}{\partial x \partial y} = f(x, y)$ then

$$\int\int_{\mathcal{R}} f(x, y) dA = F(b, d) - F(a, d) - F(b, c) + F(a, c),$$

where $\mathcal{R} = [a, b] \times [c, d]$.

3. Problem # 3 (#33 in Section 15.2)

First sketch the domain of integration specified by the integral below, then change the order of integration and evaluate the integral:

$$\int_0^1 \int_y^1 \frac{\sin(x)}{x} dx dy.$$

4. Problem # 4 (#59 in Section 15.4)

Prove the formula

$$\int\int_{\mathcal{D}} \ln(r) dA = -\frac{\pi}{2}$$

where $r = \sqrt{x^2 + y^2}$ and $\mathcal{D}$ is the unit disk $x^2 + y^2 \leq 1$. This is an improper integral since $\ln(r)$ is not defined at $(0, 0)$, so integrate first over the annulus $a \leq r \leq 1$, where $0 < a < 1$, and let $a \to 0$.

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