Math 13 Winter 20, Practice Exam I

Name: ______________________________

INSTRUCTIONS

This is a closed book, closed notes exam.

There are 9 problems. You have 2 hours.

Use of calculators is not permitted.

On each question you must show your work. No credit is given for solutions without supporting calculations.

The Honor Principle requires that you neither give nor receive any aid on this exam.

GOOD LUCK!

(1) Let f be a function of three variables x, y and z. For each of the following assertions, select the correct ending.

(a) The integral $\int_0^{2t} \int_0^{x} \int_0^{xz} f(x,y,z)\, dy\, dz\, dx$ is...

☐ a constant.

☐ a function of t.

☐ a function of y.

☐ not well-defined.

☐ none of the above.

(b) The integral $\int_0^{1} \int_x^{0} \int_0^{xz} f(x,y,z)\, dy\, dx\, dz$ is...

☐ a constant.

☐ a function of x and z.

☐ a function of y.

☐ not well-defined.

☐ none of the above.

(c) The integral $\int_0^{1} \int_0^{2} f(x,y,z)\, dx\, dz$ is...

☐ a constant.

☐ a function of x and z.

☐ a function of y.

☐ not well-defined.

☐ none of the above.

(2) Calculate the double integral

$$\iint_{\mathcal{R}} \left(\frac{x}{y} + \frac{y}{x} \right) \, dA$$

where $\mathcal{R} = [1, 4] \times [1, 2]$.

(3) Evaluate the following integral:

$$\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy.$$

(4) Calculate the triple integral $\int_0^{\sqrt{\pi}} \int_0^{x} \int_0^{xz} x^2 \cos(y)\, dy\, dz\, dx.$

(5) Find the volume of the solid that lies below the surface $z^2 = 9x^2 + 9y^2$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 2y$.

Hint: you may use without proof any of the following facts:

$$\int_0^{\frac{\pi}{2}} \sin^3\theta \, d\theta = \frac{2}{3} \quad , \quad \int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3} \quad , \quad \int_0^{\frac{\pi}{2}} \sin^2\theta \, d\theta = \frac{\pi}{4} \quad , \quad \int_0^{\pi} \sin^2\theta \, d\theta = \frac{\pi}{2}.$$

(6) The volume of the solid bounded by the surfaces with equations

$$y = x^2 + \frac{z^2}{4} \qquad \text{and} \qquad y = 5 - 4x^2 - z^2$$

is given by an integral of the form

$$\int_a^b \int_{u_1(z)}^{u_2(z)} \int_{x^2+\frac{z^2}{4}}^{5-4x^2-z^2} dy\,dx\,dz.$$

Determine $u_1(z)$, $u_2(z)$, a and b.

(7) Let $\mathcal{W}$ be the solid region lying:

- inside the cone $z = \sqrt{x^2 + y^2}$
- inside the sphere $x^2 + y^2 + z^2 = 5$
- outside the sphere $x^2 + y^2 + z^2 = 3$
- in the **second** octant: $\boldsymbol{x \leq 0}$, $y \geq 0$, $z \geq 0$.

Convert the integral $\iiint_{\mathcal{W}} x\, dV$ into a triple integral in spherical coordinates.

Do not evaluate the integral.

(8) Express the volume of the region bounded by the surfaces with equations

- $z = x^2 - 1$
- $x = y^2$
- $z = 0$

by a triple integral in the prescribed orders.

Do not evaluate the integrals.

(a) $dz\, dy\, dx$

(b) $dy\, dx\, dz$

(9) A lamina has the shape of a half-disk of radius 2, centered at the origin and lying in the upper-half plane. Its density function is $\delta(x,y) = k\sqrt{x^2+y^2}$, where k is a constant. The center of mass of the lamina has coordinates $(0, y_0)$. Determine y_0.

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