Math 13 Winter 20, Practice Exam II

Name: ______________________________

INSTRUCTIONS

This is a closed book, closed notes exam.

There are 9 problems. You have 2 hours.

Use of calculators is not permitted.

On each question you must show your work. No credit is given for solutions without supporting calculations.

GOOD LUCK!

(1) (*9 points*) For each of the following assertions, select the correct ending.

(a) Let $\mathbb{D}$ be the disk with radius 1 and center at the origin in the uv-plane and G the mapping defined by

$$G(u, v) = (u + 2v, -u + 4v).$$

The area of $G(\mathbb{D})$ is...

☐ 6π.

☐ $\frac{\pi}{6}$.

☐ π.

☐ 2π.

☐ $\frac{\pi}{2}$.

☐ none of the above.

(b) Let $f(x, y, z) = y^3$. Then $\text{grad}\,(\text{div}\,(\text{grad}\, f)) = \ldots$

☐ $6\mathbf{j}$.

☐ 6.

☐ $6y$.

☐ $\langle 0, 3y^2, 0\rangle$.

☐ none of the above.

(c) Let $\mathbf{F}$ be a vector field. Then $\text{grad}\,(\text{curl}\,\mathbf{F})$ is...

☐ a potential for $\mathbf{F}$.

☐ the divergence of $\mathbf{F}$.

☐ a vector field.

☐ not well-defined.

☐ none of the above.

(2) (*8 points*) Determine the image of the triangular region with vertices $(0,0)$, $(1,1)$ and $(0,1)$ under the transformation $G(u,v) = (u^2, v)$.

Your answer must contain two figures: the original region and its image, each in the appropriate coordinate plane with all the pertinent information.

(3) (*10 points*) The mapping $G(u,v) = \left(\frac{1}{2}(u+v), \frac{1}{2}(u-v)\right)$ transforms the trapezoidal region $\mathcal{S}$ with vertices $(-2,2)$, $(2,2)$, $(1,1)$ and $(-1,1)$ in the uv-plane into the trapezoidal region $\mathcal{R}$ with vertices $(1,0)$, $(2,0)$, $(0,-2)$ and $(0,-1)$ in the xy-plane.

Use this change of variables to evaluate the integral $\iint_{\mathcal{R}} e^{\frac{x+y}{x-y}}\, dA$.

(4) (*6 points*) The volume of the solid ellipsoid $\mathcal{E}$: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} \leq 1$ is given by:

$$\text{Vol}(\mathcal{E}) = \iiint_{\mathcal{E}} 1\, dV.$$

Use one of the following changes of variables and the fact that the ball with center at the origin and radius 1 has volume $\frac{4}{3}\pi$ to determine $\text{Vol}(\mathcal{E})$.

$$G_1(u, v, w) = \left(\frac{u}{a}, \frac{v}{b}, \frac{w}{c}\right)$$
$$G_2(u, v, w) = (au, bv, cw)$$
$$G_3(u, v, w) = (\sqrt{u}, \sqrt{v}, \sqrt{w})$$

(5) (*8 points*) Evaluate the integral

$$\int_{\mathcal{C}} 18y^3\, ds$$

where $\mathcal{C}$ is the curve in the plane parameterized by $x(t) = t^3$, $y(t) = t$ with $0 \leq t \leq 1$.

Hint: You may use without proof any of the following facts:

$$\int_0^1 \sqrt{u}\, du = \frac{2}{3} \quad , \quad \int_0^9 \sqrt{u}\, du = 18 \quad , \quad \int_1^{10} \sqrt{u}\, du = \frac{2(10\sqrt{10} - 1)}{3} \quad , \quad \int_1^9 \sqrt{u}\, du = \frac{52}{3}$$

(6) (*8 points*) Evaluate the integral

$$\int_{\mathcal{L}} xe^{yz}\, ds$$

where $\mathcal{L}$ is the line segment from $(0, 0, 0)$ to $(1, 2, 3)$.

(7) (*7 points*) Evaluate the integral of the vector field

$$\mathbf{F}(x,y,z) = \left(x + y + \frac{z}{4}\right)\mathbf{i} + (y - x^3)\mathbf{j} + \ln\left(\frac{x+z}{y+1}\right)\mathbf{k}$$

along the curve $\mathbf{r}(t) = t\mathbf{i} + t^3\mathbf{j} + 4\mathbf{k}$ for $0 \le t \le 1$.

(8) (*7 points*) The vector field $\mathbf{F}(x, y) = (3 + 2xy)\mathbf{i} + (x^2 - 3y^2)\mathbf{j}$ is conservative (**do not prove it**). Find a potential for $\mathbf{F}$.

(9) (*7 points*) Let $f(x, y) = xe^y$ and $\mathbf{F} = \nabla f$. Evaluate $\int_\Gamma \mathbf{F} \cdot d\mathbf{r}$, where Γ is given by

$$\mathbf{r}(t) = te^t\mathbf{i} + \sqrt{1+3t}\,\mathbf{j}$$

for $0 \le t \le 1$.

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