

Introduction

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Prerequisite: Math 8

OpenStax - Calculus Volume 3

Calculus Early Transcendentals - Multivariable by
Rogawski- Adams

Grading:

20% - webwork: Opens 8 am on lecture days, due 11:59pm two lecture days later. First due Monday.

20% - Written homework: Due Thursdays at 11:59 pm, covers the material from the previous week. First due 1/13, covering 1.5 and 1.7. Submit online through gradescope.

30% - Midterm, 2/08. In-person, no notes, no calculators, no collaboration. Submitted online via gradescope.

30% - Final. Date TBA. Same rules as the mid-term.

Tutorials - Everyone is split into groups in Canvas. Times TBD, ~~can~~ can be switched later.

Other - View the course website or canvas for more information about course policies and accessibility.

Masks are required. Please let us know if you are feeling ill and cannot come to class.

Review: We will need to use vectors and ~~compute~~ their properties.

Recall that $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, or

$$d_x b_x + d_y b_y + d_z b_z$$

$|\mathbf{a}|$ represents the length of \mathbf{a} , i.e. $\sqrt{a_x^2 + a_y^2 + a_z^2}$. Given a vector \mathbf{a} which is not $\mathbf{0}$, $\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$.

The projection of \mathbf{b} in the direction of \mathbf{a} is $\mathbf{b} \cdot \hat{\mathbf{a}}$.

Given two vectors \mathbf{a} and \mathbf{b} in \mathbb{R}^3 which are not parallel, we can get a third vector perpendicular to both via:

$$\mathbf{a} \times \mathbf{b} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{pmatrix} = (d_y b_z - b_y a_z) \hat{i} - (d_x b_z - b_x a_z) \hat{j} + (a_x b_y - b_x a_y) \hat{k}$$

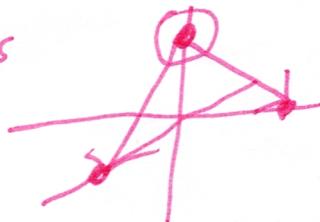
Notice that there is a negative in front of the \hat{j} term. This is always there.

Unit normal vector

We can use the cross product to find the equation of a plane containing two vectors and a point.

Ex: Give the equation of the plane containing the points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

Sol: This plane contains the point $(0, 0, 1)$ and the vectors $\langle 1, 0, -1 \rangle$, $\langle 0, 1, -1 \rangle$. Thus the normal vector is



$$\langle 1, 0, -1 \rangle \times \langle 0, 1, -1 \rangle = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} =$$
$$\hat{i} - (-1)\hat{j} + \hat{k} = \langle 1, 1, 1 \rangle.$$

Thus the plane equation is of the form $x+y+z=d$. To solve for d , we plug in the point $(0, 0, 1)$:

$$0+0+1=d, \text{ so } d=1.$$

the plane equation

We will also need to know how to take partial derivatives. Recall that this is done by treating all other variables as constants.

Ex: $\frac{d}{dx} 5e^{5x} = 25e^{5x}$

• $\frac{\partial}{\partial x} ye^{xy} = y^2 e^{xy}$

• $\frac{d}{dx} xe^{3x} = e^{3x} + 3xe^{3x}$

• $\frac{\partial}{\partial y} ye^{xy} = e^{xy} + xye^{xy}$

The gradient vector for a multivariable function, ∇f , is $\langle f_x, f_y, f_z \rangle = \langle \frac{\partial}{\partial x} f, \frac{\partial}{\partial y} f, \frac{\partial}{\partial z} f \rangle$.

The critical points of f are where the gradient vector is $\vec{0}$ or undefined.

Ex: Find the critical points of $f(x, y, z) = x^2 + 2y^3 + z^2$

so $\nabla f = \langle 2x, 6y^2, 2z \rangle$.

Thus the only critical point is $(0, 0, 0)$.