Math 14 Exam Info

Date, Time, Location: Wednesday, October 13, 2004. 7 - 9pm. 102 Bradley.

Sections Covered: 1.1 - 1.5, 2.1 - 2.6, 3.1 - 3.5

Topics Covered:

Vectors in $\mathbb{R}^n$: Vector algebra: addition, subtraction, scalar multiplication, dot and cross product. Cauchy-Schwarz and Triangle inequalities. Angle between two vectors: relation to dot and cross product. Parametric equations of lines. Normal vectors and equations of planes.

Matrices: Matrix-vector multiplication: definition of $A\mathbf{x}$, required compatibility of dimensions. Matrix-matrix multiplication: definition of AB, required compatibility of dimensions. Linear functions: definition, basic properties.

Functions of several variables:

- **Real-valued and vector-valued functions:** Relationship to each other. Level sets of real-valued functions (level curves in $\mathbb{R}^2$, level surfaces in $\mathbb{R}^3$).
- **Limits:** $\epsilon - \delta$ definition of limits (be able to use it to verify limits).
- **Continuity:** Definition in terms of limits. Properties (adding, multiplying, dividing, composing; be able to argue where a function is continuous and why).
- **Differentiation:** Partial derivatives. Iterated partial derivatives. Equality of mixed partials (what conditions are needed?). The derivative matrix (differential) and general differentiability: definition, differentiability implies continuity, C^1 implies differentiability, properties (differentiating sums, products, quotients), the chain rule.
- **Paths:** Differentiating. Velocity vectors. Tangent lines.
- **Gradients and directional derivatives:** Definitions. Relationship between the two. Interpretations. Orthogonality of level sets and gradients.
- **Taylor's Theorem:** First order and second order Taylor approximations: definitions, the size of the remainders.

- **Extrema of Real-Valued Functions:** Definitions of local/global maxima/minima and saddle points. Critical points. The Hessian. First derivative test (functions of n variables). Second derivative test: positive/negative/indefiniteness of Hessian for functions of n-variables, more explicit version for functions of 2 variables (p 216, Theorem 6). Finding and identifying local extrema on open sets. Finding and identifying global extrema on closed and bounded sets of the form $U \cup \partial U$. Finding global extrema subject to constraints: Lagrange multipliers.

- **Inverse and Implicit Function Theorem:** Be able to use these theorems to prove solvability of equations/systems of equations near given points.

Additional Info:

The exam will include both computational and theoretical (more abstract) questions. I don't yet know the exact proportions, but I would expect more of the former than the latter. There will definitely be one $\epsilon - \delta$ limit problem, so be ready!

Use of notes, books, etc. will not be allowed during the exam. You will not need a calculator nor will you be permitted to use one.