Math 14 Final Exam Topics

December 1, 2004

1 Vectors

Vector Algebra: Know the basics of vectors. Know how to compute dot and cross products. Know the formulas for the dot and cross products in terms of lengths and angles. Know the Cauchy-Schwarz inequality. Know the geometric interpretation of dot and cross products. Be able to compute planes from normal vectors and vice versa.

2 Differentiation

Limits: I reserve the right to include an $\epsilon - \delta$ problem on the exam. But in general, just be able to give a reasonable argument as to the value of a limit.

Partial Derivatives: Know how to compute them. This requirement will probably just be implicit in the computations involved in more complicated problems. Know the theorem on equality of mixed partials.

(Total) Derivatives: This is the derivative matrix (differential). Be able to compute it. Be able to tell when a function is differentiable (i.e. know that $C^1 \Rightarrow$ differentiable).

The Chain Rule: Know what it says. Keep in mind that this will require you to remember how to multiply matrices. Be able to use it to compute partial derivative of compositions.

Gradients and Directional Derivatives: Know their definitions. Know that differentiable functions have directional derivatives in all directions. Know the relationship between the gradient and the directional derivative. Know the relationship between level curves/surfaces and the gradient. Be able to compute tangent planes/normal vectors to level surfaces.

3 Maxima and Minima of Real-Valued Functions

Local Extrema: Know what they are. Know what a critical point is. Be able to find and identify them (i.e. know the first and second derivative tests).

Global Extrema: Know what they are and when they exist. Be able to find and identify them (see the box on page 221).

Constrained Extrema: Know what this means. Be able to use Lagrange multipliers. In particular, be able to use Lagrange multipliers to find global extrema.

4 Vector-Valued Functions

Curves: Know what a curve is. Know how to compute the velocity vector and speed of a particle moving on a curve. Know how to compute arc-length. Be able to parametrize familiar curves.

Vector Fields: Know what they are. Know how to compute divergence and curl. Know how to tell when a vector field is conservative. Be able to find the potential function for a onservative field. Know that $\nabla \times \nabla f = \mathbf{0}$ and $\nabla \cdot (\nabla \times \mathbf{F}) = 0$ (and, more importantly, know what kind of objects f and $\mathbf{F}$ are in these equations).

5 Integration

Double and Triple Integrals: Be able to set them up as iterated integrals and compute them. Be able to change the order of integration. Know the change of variables theorem for double integrals and be able to use it. Have some idea of what an improper integral is and how one computes them.

Path and Line Integrals: Know the difference between the two and how to compute them (i.e. how to write them as ordinary single variable integrals). Be able to parametrize familiar curves. Know the relation of path integrals to arc length. Know the relationship between line integrals and conservative vector fields. Know what orientation of a curve is.

Surface and Flux Integrals: Know the difference between the two and how to compute them (i.e. how to write them as double integrals). Know how to compute the tangent and normal vectors to a parametrized surface. Know the relationship of surface integrals to surface area. Be able to parametrize familiar surfaces. Know what orientation of a surface is.

Green's, Stokes' and Gauss' Theorems: Know the statements of these theorems. Be able to apply them in various situations (e.g. be able to use them, when appropriate, to compute a surface integral by computing a related line integral). Know how to use Green's theorem to compute the area of a planar region.