Math 14 Fall 2004

Calculus of Vector-valued Functions, Honors

Second Project: Green's Theorem and Harmonic Functions

Due Date: Wednesday, December 1, 2004

Your name:

Instructions: Solve each of the 9 problems. You must justify all of your answers to receive credit.

You will need to use your own paper. Please write **neatly** on only one side of each sheet of paper.

The Honor Principle requires that you work on this project only with other students enrolled in this class. The only other person you may consult regarding this project is the course instructor. Each student is responsible for independently writing up his or her own solutions. No copying!

In this project you will derive some of the basic properties of harmonic functions via Green's Theorem. Actually, you will need to know Green's Theorem in the form of the Divergence Theorem in the Plane. See pages 527 - 528 for the statement and the proof. You will also need to remember the Laplace operator ∇^2. For a C^2 function $f : \mathbb{R}^2 \to \mathbb{R}$, it is defined by

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}.$$

Finally, recall that a C^2 function $f : \mathbb{R}^2 \to \mathbb{R}$ is called harmonic if it satisfies Laplace's equation: $\nabla^2 f = 0$.

So here's the project: do problems 21 through 29 on pages 530 and 531. This material is very standard, so if you're stuck feel free to browse through a book on partial differential equations. However, the point of the project is for you to learn something about harmonic functions, so please don't just copy proofs from other books. If you do use another book as a resource, please cite it appropriately.