The Flow Lines of a Particular Vector Field

Problem $\star$. Let
$$\mathbf{F}(x, y) = (x^2 - y^2, 2 * x * y).$$

It was remarked in class that the flow lines of this vector field are circles tangent to the x-axis with centers on the y-axis. We'll verify this fact here.

a. Show that for any $r > 0$ the path

$$\mathbf{c}(t) = \left(\frac{-4r^2 t}{4r^2t^2 + 1}, \frac{2r}{4r^2t^2 + 1} \right)$$

is a flow line for $\mathbf{F}$.

b. Show that the image of $\mathbf{c}(t)$ lies on a circle with center $(0, r)$ and radius r (that is, $\mathbf{c}(t)$ is one of the circles mentioned earlier).

c. Can you describe how the circle in part (b) is traced out as t varies? What happens as $t \to \pm\infty$?