Review Sheet for Final Exam, Math 19 Fall 2010

I do not promise this will exactly enumerate every topic that is or is not on the exam, but it hits the vast majority and any omissions are unintentional.

Chapters 1 and 2: Logic and Sets

This will be mostly in service of later material, as vocabulary. However, I am interested in a few things by themselves: determining truth values of implications, domain of quantification to make a statement true or false, and the effect of order of existential and universal quantifiers on truth value. I will not require you to make either truth tables or Venn diagrams, but there may be instances where those are helpful. Also, you should understand power set, Cartesian product, subset, and disjointness; while I don't anticipate asking about any just on its own, all will almost certainly make an appearance in another context.

Chapters 2 and 3: Sums and Asymptotics

As for Midterm 1. You should know the sum of the numbers 1 through n and the geometric sum formula; the other two finite sum formulas on p. 157 will be provided. Be able to manipulate starting index values (to start somewhere other than 0 or 1 in the terms) and put together or take apart sums via addition and constant multiplication. Know what it means for a function to be big-O or big-Θ of another function and be able to prove or disprove that relationship. Be able to treat those formally as relations.

Chapters 2 and 8: Functions and Relations

We'll deal with binary relations only, and the properties of:
being a function which may additionally be injective or surjective
being reflexive, symmetric, antisymmetric, or transitive.

You should know the definition of equivalence relation, be able to confirm or disprove each of the properties above for a given relation, and understand relations as subsets of Cartesian products. Note that I am going to be most interested in your ability to treat things formally (as in the REF problem on Midterm 1) and in mix-and-match problems, where relations define or are defined from something else in the course (as in the in-class examples on adjacency matrices for relation digraphs) or relation properties are applied outside their normal scope (as in the "transitive function" problem on Midterm 1).

Chapter 4: Induction and Recursion

Be able to do induction for arithmetic formulas and recursively-defined functions like the Fibonacci numbers, as for Midterm 2. If I can find suitable induction problems for relations or graphs, I will prioritize them over other induction problems.

Chapters 5 and 6: Counting and Probability

The most basic formula is the multiplication principle, but you should also know the answers to the following questions cold:

(1) How many subsets does a set of size n have? 2^n.
Sample guises: sequences of n coin flips (subset is positions of Hs); subgraphs with a given set of vertices (where n is the number of edges incident to those vertices in the full graph); size of $\mathcal{P}(A)$ where $|A| = n$

(2) How many subsets of size k does a set of size n have? $\binom{n}{k}$, a.k.a. $C(n, k)$.
Sample guises: sequences of n coin flips with k heads; ways to select k out of n people for a group with no additional distinctions; ways to order a collection of two types of indistinguishable objects, k of which are one type and $n - k$ of which the other type; hands of k cards from a deck of size n

(3) How many ways are there to choose an ordered list of k elements from a set of size n? $P(n, k)$
Sample guises: ways to seat k out of n people along one side of a dinner table; ways to lay out k cards from a deck of size n on a table; possible 1st, 2nd, and 3rd place awards in a competition ($k = 3$, $n =$ number of competitors)

Be able to make probability fractions using those methods, plus the 2-set inclusion-exclusion principle. You should be able to find probabilities from any sufficient collection of plain, conditional, and intersection probabilities; this could mean understanding Bayes' Theorem thoroughly or using probability trees.

Be able to determine whether events and random variables are independent. There will be no problems on Chebyshev's inequality, but you should be able to find expected value and variance, and understand what they represent. Remember the shortcuts of adding expected value and, if the variables are independent, variance.

Chapters 9 and 10: Graphs and Trees

Know what it means for a graph to be simple, bipartite, complete, connected, or acyclic (no circuits as subgraphs) and be able to determine whether a graph has any of those properties. Know the definition of subgraph, subtree, union of graphs, simple circuit, simple path, connected component, tree, rooted or m-ary tree, complete n-vertex graph K_n, n-cycle C_n, and complete bipartite graph $K_{n,m}$. Know what it means for two vertices to be adjacent and the relationship between (in- or out-)degree sums and number of edges in a graph. Be able to produce and interpret adjacency matrices for directed and undirected graphs, whether simple or not. I am especially interested in graphs arising in other contexts from the class (which basically means relations).

For trees specifically, know Theorems 2 and 3 on p. 690 about vertices, edges, and internal vertices. Know the definitions of height, level, leaf, parent, child, spanning tree.

You do not need to know weighted graphs or trees, shortest paths, depth-first versus breadth-first searches, or minimal spanning trees. This section of the course will be weighted more heavily than the others comparative to the amount of material, but will still comprise less than a third of the exam.