1. An example showing REF is not reflexive.

Recall $\text{REF} = \{(R, S) : R, S \text{ both reflexive}\}$ for R, S relations on some fixed set A.

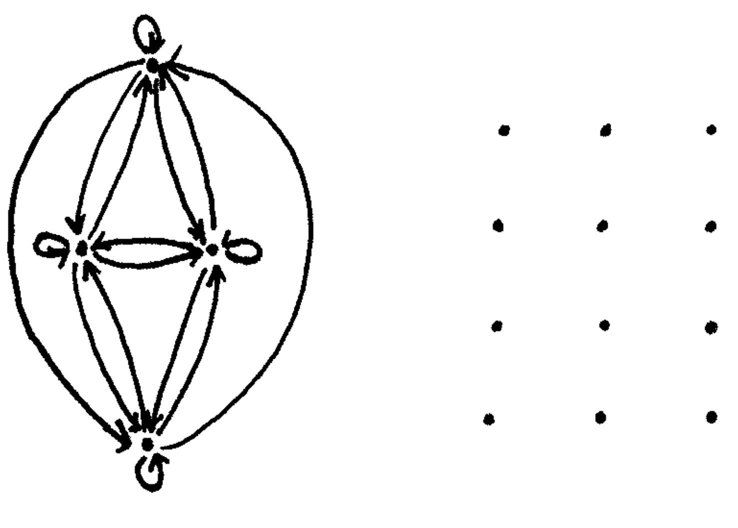
Let $A = \{1, 2\}$, so $A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$.

All possible binary relations on A:

$R_1 = \emptyset$
$R_2 = \{(1,1)\}$
$R_3 = \{(1,2)\}$
$R_4 = \{(2,1)\}$
$R_5 = \{(2,2)\}$
$R_6 = \{(1,1), (1,2)\}$
$R_7 = \{(1,1), (2,1)\}$
$R_8 = \{(2,2), (1,2)\}$
$R_9 = \{(2,2), (2,1)\}$
$R_{10} = \{(1,2), (2,1)\}$
$R_{11} = \{(1,1), (1,2), (2,1)\}$
$R_{12} = \{(2,2), (1,2), (2,1)\}$
$R_{13} = \{(1,1), (2,2)\}$
$R_{14} = \{(1,1), (2,2), (1,2)\}$
$R_{15} = \{(1,1), (2,2), (2,1)\}$
$R_{16} = A \times A$

Only R_{13} through R_{16} are reflexive.

The graph for REF defined from this A looks like this:



R_{13} through R_{16} $\qquad$ R_1 through R_{12}

2. Transitive functions on $A = \{1, 2, 3, 4, 5\}$.

For f a function from A to itself, membership of the pair (x, y) in the relation defined by f can be written simply as "$f(x) = y$". Rewriting the logical formula for transitivity with that we get

$$\forall x, y, z \in A\,((f(x) = y \;\wedge\; f(y) = z) \;\rightarrow\; f(x) = z)\,.$$

The only way f can satisfy this and still be a function is if $y = z$.

Therefore, if $f(x) = y$, we must also have $f(y) = y$.

Possible solutions:

- The identity function: if $f(x) = x$, certainly $f(x) = x$.
- Any constant function: $f(x) = f(y)$ for all x, y.
  (5 possibilities for this A)

Both of the above are examples of the most general solution:

Partition A and for each partition set $X \subseteq A$, define f as a constant function from X to X.

Example: $f(1) = f(3) = 1$; $f(2) = f(5) = 5$; $f(4) = 4$.

If you partition A into 5 sets, this gives the identity function, and if you partition A into 1 set, this gives one of the constant functions.