Mathematics 19
Introduction to Set Theory
Winter 2020
Homework Assignments

Week 1

Homework assigned Monday, January 6:

Reading (due Wednesday, January 8): Read Chapter 1, Introduction, pages 1-16. Do exercise 7 on page 9.

Written homework (due Monday, January 13): Exercises 2, 3, and 4 on page 7.

Proof-writing homework (due Monday, January 13): Exercise 35 on page 34.

Homework assigned Wednesday, January 8:

Reading (due Friday, January 10): Read Chapter 2, pages 17-23. Answer the following question: Suppose A is a set. Do our axioms guarantee that $\{\{b\} \mid b \in A\}$ is a set?

Written homework (due Monday, January 13): Exercise 5 on page 9. Also the following exercise:

> A set A is defined to be *transitive* if every element of A is also a subset of A, that is,
>
> $$\forall x(x \in A \implies x \subseteq A).$$
>
> Prove that for every transitive set A, the power set $\mathcal{P}A$ is also transitive.

No new proof-writing homework.

Homework assigned Friday, January 10:

Reading (due Monday, January 12): Read Chapter 2, pages 23-27. Do the following exercise:

> Suppose $rank(A) = 10$. What is $rank(\bigcup A)$? (The notion of rank was defined, somewhat informally, in Chapter 1.)

Written homework (due Monday, January 13): Exercises 18 and 19 on page 32.

No new proof-writing homework.

Week 2

Homework assigned Monday, January 13:

Reading (due Wednesday, January 15): Read Chapter 2, pages 27-34. Do exercise 29 on page 33.

Written homework (due Tuesday, January 21): Exercises 2, 6, and 8 on page 26.

Proof-writing homework (due Tuesday, January 21): Do the following exercise:

> Recall that a set A is transitive if every element of A is also a subset of A. Also recall $\mathbb{N} = \{0, 1, 2, \ldots, n, \ldots\}$ is the set of natural numbers, which includes 0. Finally, note that by $\omega + 0$ we mean ω.
>
> Show that, for every $n \in \mathbb{N}$, the sets V_n and $V_{\omega+n}$ defined on pages 7-8 are transitive. Assume there are no atoms, so $V_0 = \emptyset$.

Homework assigned Wednesday, January 15:

Reading (due Friday, January 17): Read Chapter 3, pages 35-42. Answer the following question: Using the idea Norbert Wiener used in his definition of ordered pair $\langle x, y \rangle = \{\{\{x\}, \emptyset\}, \{\{y\}\}\}$, how might you define the ordered triple $\langle x, y, z \rangle$?

Written homework (due Tuesday, January 21): Exercises 25 and 32 on pages 33-34.

No new proof-writing homework.

Homework assigned Friday, January 17:

Reading (due Tuesday, January 21): Read Chapter 3, pages 42-55. Do exercise 13 on page 53.

Written homework (due Tuesday, January 21): Exercises 2, 4, and 5 on pages 38-39.

No new proof-writing homework.

Week 3

Homework assigned Tuesday, January 21:

Reading (due Wednesday, January 22): Read Chapter 3, pages 55-62.

Written homework (due Monday, January 27): Exercises 14, 18, and 27 on page 53.

No new proof-writing homework.

Homework assigned Wednesday, January 15:

Reading (due Friday, January 24): Read Chapter 3, pages 62-66. Do exercise 43 on page 64

Written homework (due Monday, January 27): Exercises 37, 40, and 41 on pages 61-62.

Proof-writing homework (due Wednesday, January 29): Do the following exercise.

> Let Q be the equivalence relation on $\mathbb{R} \times \mathbb{R}$ defined in exercise 41 on page 62. Show there is a function $F : (\mathbb{R} \times R)/Q \to (\mathbb{R} \times R)/Q$ such that $F([\langle a, b\rangle]_Q) = [\langle a^2 + b^2, 2ab\rangle]_Q$.

Homework assigned Friday, January 17:

Reading (due Monday, January 27): Read Chapter 4, pages 67-73.

Written homework (due Monday, January 27): Exercises 44 and 45 on page 64.

No new proof-writing homework.

Week 4

Homework assigned Monday, January 27:

Reading (due Wednesday, January 29): Read Chapter 4, pages 73-89.

Written homework (due Monday, February 3): Exercises 5(a) on page 73 and 8 on page 78.

No new proof-writing homework.

Homework assigned Wednesday, January 29:

Reading (due Monday, February 3): Read Chapter 5, pages 90-101. Do exercise 1 on page 101.

Written homework (due Monday, February 3): Exercises 9 and 10 on page 78 and 26 on page 88.

No new proof-writing homework.

Homework assigned Friday, January 31:

No new homework.

Week 5

Homework assigned Monday, February 3:

Reading (due Wednesday, February 5): Read Chapter 5, pages 123-127.

Written homework (due Monday, February 10): Exercises 15 on page 83 and 18 and 19 on page 88.

Proof-writing homework (due Monday, February 10): Do the following exercise.

> Give a formula for subtraction of integers:
>
> $$[\langle m, n\rangle] - [\langle p, q\rangle] = ?$$
>
> Show this operation is well defined.
>
> Show that subtraction is the inverse operation for addition: For all integers x and y,
>
> $$(x - y) + y = x.$$

Homework assigned Wednesday, February 5:

Reading (due Friday, February 7): Read Chapter 6, pages 128-133. Do exercise 2 on page 133.

Written homework (due Monday, February 10): Exercises 4 and 6 on page 101.

No new proof-writing homework.

Homework assigned Friday, February 7:

Reading (due Monday, February 10): Read Chapter 6, pages 133-138. Do exercise 6 on page 138.

Written homework (due Monday, February 10): Exercises 4, and 5 on page 133.

No new proof-writing homework.

Week 6

Homework assigned Monday, February 10:

Reading (due Wednesday, February 12): Read Chapter 6, pages 138-145. Do exercise 10 on page 144.

Written homework (due Monday, February 17): Exercises 7 and 8 on page 138.

Proof-writing homework (due Monday, February 17): Do the following exercise.

> Prove that for any natural number n and any set X, either $card(X) \in n$, or there is a subset $Y \subseteq X$ with $card(Y) = n$.
>
> Conclude that if X is an infinite set, then X has subsets of every finite cardinality.
>
> Note 1: You should not use results beyond page 138 of the textbook. You may use the results in the section on finite sets.
>
> Note 2: Be formal and careful with your proof. As a warning: You might think that if X has no finite subsets, then X must have a subset equinumerous with ω. However, this cannot be proven without the Axiom of Choice. (You will not need to use the Axiom of Choice for the proof you are asked to do here.)

Homework assigned Wednesday, February 12:

Reading (due Friday, February 14): Read Chapter 6, pages 145-151.

Written homework (due Monday, February 17): Exercises 12 and 13 on page 144.

No new proof-writing homework.

Homework assigned Friday, February 14:

Reading (due Monday, February 17): Read Chapter 6, pages 151-159. Do exercise 23 on page 158.

Written homework (due Monday, February 17): Exercises 15, 16, and 17 on pages 150-151.

No new proof-writing homework.

Week 7

Homework assigned Monday, February 17:

Reading (due Wednesday, February 19): Read Chapter 6, pages 159-166.

Written homework (due Monday, February 24): Exercises 19 and 22 on page 158.

No new proof-writing homework.

Homework assigned Wednesday, February 19:

Reading (due Monday, February 24): Read Chapter 7, pages 167-179. Do exercise 1 on page 172.

Written homework (due Monday, February 24): Exercises 26 on page 161 and 32 and 34 on page 165.

No new proof-writing homework.

Homework assigned Friday, February 21:

No new homework.

Week 8

Homework assigned Monday, February 24:

Reading (due Wednesday, February 26): Read Chapter 7, pages 179-187.

Written homework (due Monday, March 2): Exercises 4 and 7 on page 178.

Proof-writing homework (due Monday, March 2): Do the following exercise.

> Recall that $Sq(A)$ is the set of all finite sequences in A, where we define a finite sequence in A to be a function from some natural number n into A. Show that if A is infinite then $A \approx Sq(A)$. You may use the Axiom of Choice.

Homework assigned Wednesday, February 26:

Reading (due Friday, February 28): Read Chapter 7, pages 187-195.

Written homework (due Monday, March 2): Exercises 10 on page 184 and 14 on page 187.

No new proof-writing homework.

Homework assigned Friday, February 28:

Reading (due Monday, March 2): Read Chapter 7, pages 195-200.

Written homework (**Change: due Friday, March 6**): Exercises 16, 17, and 19 on pages 194-195.

No new proof-writing homework.

Week 9

Homework assigned Monday, March 2:

Reading (due Wednesday, March 4): Read Chapter 7, pages 200-208.

Written homework (due Friday, March 6): Exercises 23 and 24 on page 200.

No new proof-writing homework.

Homework assigned Wednesday, March 4:

No new reading homework.

Written homework (due Friday, March 6): Exercises 29 and 30 on page 207.

No new proof-writing homework.