1. Find the derivative of the following functions:

   (a) $y = x^e$

   (b) $g(x) = 1.6^x + x^{1.6}$

   (c) $f(t) = \pi^{-t}$

2. For each of the following, find the equation of the tangent line to the given curve at the given point.

   (a) $y = \frac{x}{x-3}$, $(6, 2)$

   (b) $y = \frac{x}{1+x^2}$, $(3, 0.3)$

   (c) $y = \frac{1}{1+x^2}$, $(-1, \frac{1}{2})$

   (d) $y = 10^x$, $(1, 10)$

3. Find the equation of the normal line to the curve $y = \frac{1}{x-1}$ at the point $(2, 1)$.

4. Find the equations of the tangent lines to the curve $y = \frac{x-1}{x+1}$ that are parallel to the line $x - 2y = 1$.

5. Find an equation of the tangent to the curve $y = e^x$ that is parallel to the line $x - 4y = 1$.

6. Find an equation of the tangent to the curve $y = e^x$ that passes through the origin.

7. If f is a differentiable function, find an expression for the derivative of each of the following functions:

   (a) $y = x^2 f(x)$

   (b) $y = \frac{f(x)}{x^2}$

   (c) $y = \frac{x^2}{f(x)}$

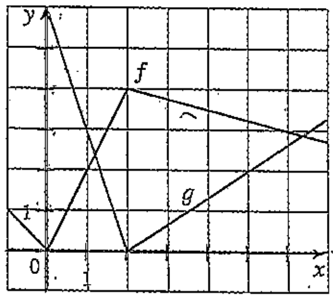
   (d) $y = \frac{1+xf(x)}{\sqrt{x}}$

8. Find f' in terms of g' if $f(x) = x^2 g(x)$.

9. Find h' in terms of f' and g' if $h(x) = \frac{f(x)g(x)}{f(x)+g(x)}$.

10. The function g is a twice differentiable function. Find f'' in terms of g, g', and g'' if $f(x) = \frac{g(x)}{x}$.

11. If f and g are functions whose graphs are shown, let $u(x) = f(x)g(x)$ and $v(x) = \frac{f(x)}{g(x)}$.



(a) Find $u'(1)$.

(b) Find $v'(5)$.

12. Find the following limits.

(a) $\lim_{x\to a} \frac{\sqrt[3]{x}-\sqrt[3]{a}}{x-a}$, $a \neq 0$

(b) $\lim_{x\to 0} \frac{6^x-2^x}{x}$

(c) $\lim_{x\to 0} \frac{e^x-1-x}{x^2}$

(d) $\lim_{x\to 0} \frac{e^x-1-x-\frac{x^2}{2}}{x^3}$

(e) $\lim_{x\to -\infty} xe^x$.

13. For each of the following functions find

- the intervals of increase or decrease,
- the local maximum or minimum values,
- the intervals of concavity, and
- the x-coordinates of the points of inflection.

(a) $f(x) = x^3 - x$

(b) $f(x) = 2x^3 + 5x^2 - 4x$

(c) $f(x) = x^4 - 6x^2$

(d) $g(x) = x^4 - 3x^3 + 3x^2 - x$

(e) $h(x) = 3x^5 - 5x^3 + 3$

(f) $Q(x) = x - 3x^{\frac{1}{3}}$

1) (a) $y = x^e$

$\underline{\underline{y' = e x^{e-1}}}$

(b) $g(x) = 1.6^x + x^{1.6}$

$\underline{\underline{g'(x) = \ln(1.6)\, 1.6^x + 1.6\, x^{0.6}}}$

(c) $f(t) = \pi^{-t}$

$f'(t) = \ln(\pi)\, \pi^{-t}(-1)$

$= \underline{\underline{-\ln(\pi)\, \pi^{-t}}}$

2) (a) $y = \frac{x}{x-3}$, $(6, 2)$

$y' = \frac{x-3-x}{(x-3)^2} = -\frac{3}{(x-3)^2}$

$y'(6) = -\frac{1}{3}$

$y = -\frac{1}{3}x + b$

$2 = -\frac{1}{3}(6) + b = -2 + b$

$4 = b$

$\underline{\underline{y = -\frac{1}{3}x + 4}}$

(b) $y = \frac{x}{1+x^2}$, $(3, 0.3)$

$y' = \frac{1+x^2 - 2x(x)}{(1+x^2)^2}$

$= \frac{1-x^2}{(1+x^2)^2}$

$y'(3) = \frac{-8}{100} = -\frac{2}{25}$

$y = -\frac{2}{25}x + b$

$\frac{3}{10} = -\frac{2}{25}(3) + b = -\frac{6}{25} + b$

$\frac{27}{50} = b$

$\underline{\underline{y = -\frac{2}{25}x + \frac{27}{50}}}$

(c) $y = \frac{1}{1+x^2}$, $(-1, \frac{1}{2})$

$y' = \frac{-2x}{(1+x^2)^2}$

$y'(-1) = \frac{2}{4} = \frac{1}{2}$

$y = \frac{1}{2}x + b$

$\frac{1}{2} = -\frac{1}{2} + b$

$1 = b$

$\underline{\underline{y = \frac{1}{2}x + 1}}$

(d) $y = 10^x$, $(1, 10)$

$y' = \ln(10)\, 10^x$

$y'(1) = 10 \ln(10) = \ln(10^{10})$

$y = \ln(10^{10})x + b$

$10 = \ln(10^{10}) + b$

$\frac{1}{10}\ln(10^{10}) = b$

$\ln(10) = b$

$\underline{\underline{y = \ln(10^{10})x + \ln(10)}}$

3) $y = \frac{1}{x-1}$

$y' = -\frac{1}{(x-1)^2}$

$y'(2) = -1$

Slope of normal line: 1

$y = x + b$

$2 = 1 + b$

$1 = b$

$\underline{\underline{y = x + 1}}$

4) $y = \frac{x-1}{x+1}$

$y' = \frac{x+1-(x-1)}{(x+1)^2} = \frac{2}{(x+1)^2}$

~~Slope of x - 2y = 1~~ $2y = x - 1$

$y = \frac{1}{2}x - \frac{1}{2}$

Slope: $\frac{1}{2}$

$\frac{2}{(x+1)^2} = \frac{1}{2}$

$\Rightarrow (x+1)^2 = 4$

$\Rightarrow x^2 + 2x - 3 = 0$

$\Rightarrow (x+3)(x-1) = 0$

$\Rightarrow x = -3, 1$

$\cancel{y'(-3) = \frac{2}{4} = \frac{1}{2}}$

$y = \frac{1}{2}x + b$

$2 = \frac{1}{2}(-3) + b = -\frac{3}{2} + b$

$\frac{7}{2} = b$

$\underline{\underline{y = \frac{1}{2}x + \frac{7}{2}}}$

$0 = \frac{1}{2}(1) + b$

$-\frac{1}{2} = b$

$\underline{\underline{y = \frac{1}{2}x - \frac{1}{2}}}$

5) $y = e^x$ $\qquad x - 4y = 1$

$y' = e^x$ $\qquad 4y = x - 1$

$\qquad y = \frac{1}{4}x - \frac{1}{4}$

$e^x = \frac{1}{4}$

$x = \ln\left(\frac{1}{4}\right) = -\ln(4)$

$y = \frac{1}{4}x + b$

$\frac{1}{4} = \frac{1}{4}(-\ln 4) + b$

$= -\frac{1}{4}\ln 4 + b$

$\frac{1}{4}(1 + \ln 4) = b$

$\underline{\underline{y = \frac{1}{4}x + \frac{1}{4}(1 + \ln 4)}}$

6) $y = e^x$

$y' = e^x$

$e^a = e^a a + b = ae^a + b$

$e^a(1-a) = b$

$y = e^a x + e^a(1-a)$

$0 = e^a(1-a)$

$\Rightarrow a = 1$ since $e^a \neq 0$ for all a

$\underline{\underline{y = ex}}$

7) (a) $y = x^2 f(x)$

$y' = \underline{\underline{2x f(x) + x^2 f'(x)}}$

(b) $y = \frac{f(x)}{x}$

$y' = \underline{\underline{\frac{f'(x)}{x} - \frac{f(x)}{x^2}}}$

(c) $y = \frac{x^2}{f(x)}$

$y' = \underline{\underline{\frac{2xf(x) - f'(x)x^2}{(f(x))^2}}}$

(d) $y = \frac{1 + xf(x)}{\sqrt{x}}$

$= \underline{\underline{\frac{(f(x) + xf'(x))\sqrt{x} - \frac{1}{2\sqrt{x}}(1 + xf(x))}{x}}}$

8) $f(x) = x^2 g(x)$

$f'(x) = \underline{\underline{x^2 g'(x) + 2x g(x)}}$

9) $h(x) = \frac{f(x)\, g(x)}{f(x) + g(x)}$

$h'(x) = \frac{(f'(x)g(x) + f(x)g'(x))[f(x) + g(x)]}{[f(x) + g(x)]^2}$

$- \frac{(f'(x) + g'(x))f(x)g(x)}{[f(x) + g(x)]^2}$

$= \underline{\underline{\frac{f'(x)g(x)^2 + g'(x)f(x)^2}{[f(x) + g(x)]^2}}}$

10) $f(x) = \frac{g(x)}{x}$

$f'(x) = \frac{g'(x)\,x - g(x)}{x^2}$

$f''(x) = \frac{(g''(x)x + g'(x) - g'(x))x^2 - 2x(g'(x)x - g(x))}{x^4}$

$= \underline{\underline{\frac{x^3 g''(x) - 2x^2 g'(x) - 2x\, g(x)}{x^4}}}$

11) $u(x) = f(x)\, g(x)$

$v(x) = \frac{f(x)}{g(x)}$

(a) $u'(x) = f'(x)g(x) + f(x)g'(x)$

$u'(1) = f'(1)g(1) + f(1)g'(1)$

$= 2(3) + 2(-3)$

$= \underline{\underline{0}}$

(b) $v'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$

$v'(5) = \frac{f'(5)g(5) - g'(5)f(5)}{[g(5)]^2}$

$= \frac{\left(-\frac{1}{4}\right)(2) - (1)\left(\frac{13}{4}\right)}{4}$

$= \frac{-\frac{1}{2} - \frac{13}{4}}{4} = -\frac{\frac{15}{4}}{4} = \underline{\underline{-\frac{15}{6}}}$

12) (a) $\lim_{x\to a} \dfrac{\sqrt[3]{x} - \sqrt[3]{a}}{x-a}$

$= \lim_{x\to a} \dfrac{\frac{1}{3}x^{-\frac{2}{3}}}{1} = \underline{\underline{\frac{1}{3}a^{-\frac{2}{3}}}}$

(b) $\lim_{x\to 0} \dfrac{6^x - 2^x}{x}$

$= \lim_{x\to 0} \dfrac{\ln(6)6^x - \ln(2)2^x}{1}$

$= \ln(6) - \ln(2)$

$= \underline{\ln(3)}$

(c) $\lim_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$

$= \lim_{x\to 0} \dfrac{e^x - 1}{2x}$

$= \lim_{x\to 0} \dfrac{e^x}{2} = \underline{\underline{\frac{1}{2}}}$

(d) $\lim_{x\to 0} \dfrac{e^x - 1 - x - \frac{1}{2}x^2}{x^3}$

$= \lim_{x\to 0} \dfrac{e^x - 1 - x}{3x^2}$

$= \lim_{x\to 0} \dfrac{e^x - 1}{6x}$

$= \lim_{x\to 0} \dfrac{e^x}{6} = \underline{\underline{\frac{1}{6}}}$

(e) $\lim_{x\to\infty} xe^x$

$= \underline{\underline{\infty}}$

13) (a) $f(x) = x^3 - x$

$f'(x) = 3x^2 - 1$

$f'(x) = 0$ for $x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$f''(x) = 6x$

$f''(x) = 0$ for $x = 0$

x-value	$f'(x)$	$f''(x)$	$f(x)$
$x < -\frac{\sqrt{3}}{3}$	+	−	incr., concave ↓
$-\frac{\sqrt{3}}{3}$			local max.
$-\frac{\sqrt{3}}{3} < x < 0$	−	−	decr., concave ↓
0			POI
$0 < x < \frac{\sqrt{3}}{3}$	−	+	decr., concave ↑
$\frac{\sqrt{3}}{3}$			local min.
$x > \frac{\sqrt{3}}{3}$	+	+	incr., concave ↑

$f(-\frac{\sqrt{3}}{3}) = \frac{\sqrt{3}}{3} - \frac{3^{\frac{3}{2}}}{27} \approx 0.385$, $f(\frac{\sqrt{3}}{3}) = \frac{3^{\frac{3}{2}}}{27} - \frac{\sqrt{3}}{3} \approx -0.385$

(b) $f(x) = 2x^3 + 5x^2 - 4x$

$f'(x) = 6x^2 + 10x - 4$

$= 2(3x^2 + 5x - 2)$

~~$= 2(3x - 2)(x + 1)$~~

$25 - 4(3)(-2)$

$= 25 + 24 = 49$

$\dfrac{-5 \pm \sqrt{49}}{6} = \dfrac{-5 \pm 7}{6} = -2, \frac{1}{3}$

$f''(x) = 12x + 10$

$f''(x) = 0$ for $x = -\frac{10}{12} = -\frac{5}{6}$

x-value	$f'(x)$	$f''(x)$	$f(x)$
$x < -2$	+	−	incr., concave ↓
-2			local max.
$-2 < x < -\frac{5}{6}$	−	−	decr., concave ↓
$-\frac{5}{6}$			POI
$-\frac{5}{6} < x < \frac{1}{3}$	−	+	decr., concave ↑
$\frac{1}{3}$			local min.
$x > \frac{1}{3}$	+	+	incr., concave ↑

$f(-2) = 12$, $f(\frac{1}{3}) = \frac{2}{27} + \frac{5}{9} - \frac{4}{3} = -\frac{19}{27}$

(c) $f(x)=x^4-6x^2$

$f'(x)=4x^3-12x$

$\quad = x(4x^2-12)$

$f'(x)=0$ for $x=0, \pm\sqrt{3}$

$f''(x) = 12x-12$

$\quad = 12(x-1)$

$f''(x)=0$ for $x=1$

x-value	$f'(x)$	$f''(x)$	$f(x)$
$x<-\sqrt{3}$	$-$	$-$	decr., concave ↓
~~$\sqrt{3}$~~ $-\sqrt{3}$			Local min
$-\sqrt{3}<x<0$	$+$	$-$	incr., concave ↓
0			local max
$0<x<1$	$-$	$-$	decr., concave ↓
1			POI
$1<x<\sqrt{3}$	$-$	$+$	decr., concave ↑
$\sqrt{3}$			Local min
$x>\sqrt{3}$	$+$	$+$	incr., concave ↑

$f(-\sqrt{3}) = 9-6(3)$

$\quad = -9$

$f(0)=0$

$f(\sqrt{3}) = -9$

(d) $g(x) = x^4-3x^3+3x^2-x$

$g'(x)=4x^3-9x^2+6x-1$

$\quad = (x-1)(4x^2-5x+1)$

$\quad = (x-1)(4x-1)(x-1)$

$g'(x)=0$ for $x=1, \frac{1}{4}$

$g''(x) = 12x^2-18x+6$

$\quad = 6(2x^2-3x+1)$

$\quad = 6(2x-1)(x-1)$

$g''(x)=0$ for $x=\frac{1}{2}, 1$

x-value	$g'(x)$	$g''(x)$	$g(x)$
$x<\frac{1}{4}$	$-$	$+$	decr., concave up
$\frac{1}{4}$		$-$	local min.
$\frac{1}{4}<x<\frac{1}{2}$	$+$	$+$	cr., concave up
$\frac{1}{2}$			POI
$\frac{1}{2}<x<1$	$+$	$-$	incr., concave down
1			POI
$x>1$	$+$	$+$	incr., concave up

$f(\frac{1}{4}) = \frac{1}{256} - \frac{3}{64} + \frac{3}{16} - \frac{1}{4} = -\frac{27}{256}$

(e) $h(x) = 3x^5 - 5x^3 + 3$

$h'(x) = 15x^4 - 15x^2 = 15x^2(x^2-1)$

$h'(x) = 0$ for $x = 0, \pm 1$

$h''(x) = 60x^3 - 30x = 30x(2x^2-1)$

$h''(x) = 0$ for $x = 0, \pm\frac{\sqrt{2}}{2}$

x-value	$h'(x)$	$h''(x)$	$h(x)$
$x<-1$	+	−	incr., concave down
-1			local max
$-1<x<-\frac{\sqrt{2}}{2}$	~~+~~ −	−	incr., concave down
$-\frac{\sqrt{2}}{2}$			POI, ~~local max~~
$-\frac{\sqrt{2}}{2}<x<0$	−	+	decr., concave up
0			POI
$0<x<\frac{\sqrt{2}}{2}$	−	−	decr., concave down
$\frac{\sqrt{2}}{2}$			POI
$\frac{\sqrt{2}}{2}<x<1$	−	+	decr., concave up
1		+	local min
$x>1$	+	+	incr., concave up

$h(-1) = -3+5+3 = 5$

$h(1) = 3-5+3 = 1$

(f) $Q(x) = x - 3x^{\frac{1}{3}}$

$Q'(x) = 1 - x^{-\frac{2}{3}} = \dfrac{x^{\frac{2}{3}}-1}{x^{\frac{2}{3}}}$

$Q'(x) = 0$ or is undefined for

$x = 0, \pm 1$

$Q''(x) = +\frac{2}{3}x^{-\frac{5}{3}} = \dfrac{2}{3x^{\frac{5}{3}}}$

$Q''(x)$ is never 0

x-value	$Q'(x)$	$Q''(x)$	$Q(x)$
$x<-1$	+	−	incr., concave down
-1		−	local max.
$-1<x<0$	−	−	decr., concave down
0			POI
$0<x<1$	−	+	decr., concave up
1		+	local min.
$x>1$	+	+	incr., concave up

$Q(-1) = -1 - 3(-1) = -1+3 = 2$

$Q(1) = 1 - 3(1) = 1-3 = -2$