

1. Find the derivative of the following functions:

$$y = \tan(3x)$$

$$y = \cos(x^3)$$

$$y = \tan(x^2) + \tan^2 x$$

$$y = \sin(\sin x)$$

$$y = \cot(\sqrt[3]{1+x^2})$$

$$y = \sin^2(\cos(4x))$$

$$y = \sin(\tan(\sqrt{\sin x}))$$

$$y = \ln(\csc(5x))$$

$$y = \arcsin\left(\frac{x-1}{x+1}\right)$$

$$g(x) = e^{-5x} \cos(3x)$$

$$y = e^{x \cos x}$$

$$y = \sec(e^{\tan(x^2)})$$

$$h(y) = \ln(y^3 \sin y)$$

$$f(x) = \arcsin(2x - 1)$$

$$y = (\arcsin x)^2$$

$$h(x) = (\arcsin x) \ln x$$

$$f(t) = \frac{\arccos t}{t}$$

$$F(t) = \sqrt{1-t^2} + \arcsin t$$

$$y = \arccos\left(\frac{b+a \cos x}{a+b \cos x}\right), 0 \leq x \leq \pi, a > b > 0$$

$$y = \arctan\left(\frac{x}{a}\right) + \ln\left(\sqrt{\frac{x-a}{x+a}}\right)$$

$$y = \frac{\sin^2 x}{\cos x}$$

$$y = 4 \sec(5x)$$

$$y = \cos^3 x$$

$$y = \cos(\tan x)$$

$$y = \sqrt{1+2 \tan x}$$

$$\sin^3 x + \cos^3 x$$

$$y = \sin\left(\frac{1}{x}\right)$$

$$y = \sqrt{\cos(\sin^2 x)}$$

$$y = \ln(\sec^2 x)$$

$$y = \arctan(\arcsin(\sqrt{x}))$$

$$h(\theta) = e^{\sin(5\theta)}$$

$$y = \tan(e^{3x-2})$$

$$f(x) = \cos(\ln x)$$

$$y = (\ln(\sin x))^3$$

$$g(x) = \arctan(x^3)$$

$$y = \arcsin(x^2)$$

$$H(x) = (1+x^2) \arctan x$$

$$g(t) = \arcsin\left(\frac{4}{t}\right)$$

$$G(t) = \arccos(\sqrt{2t-1})$$

$$f(x) = \arccos(\arcsin x)$$

2. Find  $y'$  and  $y''$  if  $y = \ln(\sec x + \tan x)$ .

3. Find  $f'$  in terms of  $g'$  if  $f(x) = g(\tan(\sqrt{x}))$ .

4. For each of the following, find the equation of the tangent line to the given curve at the given point.

(a)  $y = \sin x + \cos(2x)$ ,  $(\frac{\pi}{6}, 1)$

(b)  $y = e^{-x} \sin x$ ,  $(\pi, 0)$

(c)  $y = 2 \sin x$ ,  $(\frac{\pi}{6}, 1)$

(d)  $y = \tan x$ ,  $(\frac{\pi}{4}, 1)$

(e)  $y = \sec x - 2 \cos x$ ,  $(\frac{\pi}{3}, 1)$

5. An object with weight  $W$  is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle  $\theta$  with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin \theta + \cos \theta},$$

where  $\mu$  is a constant called the *coefficient of friction*. Find the rate of change of  $F$  with respect to  $\theta$ .

6. For each of the following curves, find the points at which the tangent line is horizontal.

(a)  $y = \sin x + \cos x$  for  $0 \leq x \leq 2\pi$

(b)  $f(x) = 2 \sin x + \sin^2 x$

(c)  $y = \sin(2x) - 2 \sin x$

(d)  $f(x) = x + 2 \sin x$

(e)  $y = \frac{\cos x}{2 + \sin x}$

7. The displacement of a particle on a vibrating string is given by the equation

$$s(t) = 10 + \frac{1}{4} \sin(10\pi t),$$

where  $s$  is measured in centimeters and  $t$  in seconds. Find the velocity of the particle after  $t$  seconds.

8. If the equation of motion of a particle is given by  $s = A \cos(\omega t + \delta)$ , the particle is said to undergo *simple harmonic motion*.

(a) Find the velocity of the particle at time  $t$ .

(b) When is the velocity 0?

9. A mass attached to a vertical spring has position function given by

$$y(t) = A \sin(\omega t),$$

where  $A$  is the amplitude of its oscillations and  $\omega$  is a constant. Find the velocity and acceleration as functions of time.

10. Find  $h'$  in terms of  $f'$  and  $g'$  if  $h(x) = f(g(\sin(4x)))$ .

11. For each of the following functions find

- the intervals of increase or decrease,
- the local maximum or minimum values,
- the intervals of concavity, and
- the  $x$ -coordinates of the points of inflection.

(a)  $f(\theta) = \sin^2 \theta$  for  $0 \leq \theta \leq 2\pi$

(b)  $f(t) = t + \cos t$  for  $0 \leq t \leq 2\pi$

$$1). \frac{d}{dx} [\tan(3x)] = 3 \sec^2(3x)$$

$$\Rightarrow \frac{d}{dx} [4 \sec(5x)] = 20 \sec(5x) \tan(5x)$$

$$\cdot \frac{d}{dx} [\cos(x^3)] = -3x^2 \sin(x^3)$$

$$\cdot \frac{d}{dx} [\cos^3 x] = -3 \cos^2 x \sin x$$

$$\cdot \frac{d}{dx} [\tan(x^2)^2 + \tan^2 x]$$

$$= \sec^2(x^2) 2x + 2 \tan x \sec^2 x$$

$$= 2x \sec^2(x^2) + 2 \tan x \sec^2 x$$

$$\cdot \frac{d}{dx} [\cos(\tan x)]$$

$$= -\sin(\tan x) \sec^2 x$$

$$\cdot \frac{d}{dx} [\sin(\sin x)]$$

$$= \cos(\sin x) \cos x$$

$$\cdot \frac{d}{dx} [\sqrt{1+2\tan x}]$$

$$= \frac{1}{2} (1+2\tan x)^{-\frac{1}{2}} (2 \sec^2 x)$$

$$= \frac{\sec^2 x}{\sqrt{1+2\tan x}}$$

$$\cdot \frac{d}{dx} [\cot(\sqrt[3]{1+x^2})]$$

$$= -\csc^2(\sqrt[3]{1+x^2})^{\frac{1}{3}} (1+x^2)^{-\frac{2}{3}} (2x)$$

$$= -\frac{2}{3} x (1+x^2)^{-\frac{2}{3}} \csc^2(\sqrt[3]{1+x^2})$$

$$\cdot \frac{d}{dx} [\sin^3 x + \cos^3 x]$$

$$= 3 \sin^2 x \cos x - 3 \cos^2 x \sin x$$

$$= 3 \cos x \sin x (\sin x - \cos x)$$

$$\cdot \frac{d}{dx} [\sin^2(\cos(4x))]$$

$$= 2 \sin(\cos(4x)) \cos(\cos(4x)) (-\sin(4x) 4)$$

$$= -8 \sin(\cos(4x)) \cos(\cos(4x)) \sin(4x)$$

$$\cdot \frac{d}{dx} [\sin(\frac{1}{x})]$$

$$= \cos(\frac{1}{x}) (-\frac{1}{x^2}) = -\frac{\cos(\frac{1}{x})}{x^2}$$

$$\cdot \frac{d}{dx} [\sin(\tan(\sqrt{\sin x}))]$$

$$= \cos(\tan(\sqrt{\sin x})) \sec^2(\sqrt{\sin x})$$

$$\cdot \frac{1}{2} (\sin x)^{-\frac{1}{2}} \cos x$$

$$= \frac{\cos x \sec^2(\sqrt{\sin x}) \cos(\tan(\sqrt{\sin x}))}{2 \sqrt{\sin x}}$$

$$\cdot \frac{d}{dx} [\sqrt{\cos(\sin^2 x)}]$$

$$= \frac{1}{2} (\cos(\sin^2 x))^{-\frac{1}{2}} (-\sin(\sin^2 x))$$

$$\cdot 2 \sin x \cos x$$

$$= -\frac{\cos x \sin x \sin(\sin^2 x)}{\sqrt{\cos(\sin^2 x)}}$$

$$\cdot \frac{d}{dx} [\ln(\csc(5x))]$$

$$= \frac{-5 \cot(5x) \csc(5x)}{\csc(5x)}$$

$$= -5 \cot(5x)$$

$$\cdot \frac{d}{dx} [\ln(\sec^2 x)]$$

$$= \frac{2 \sec x \sec x \tan x}{\sec^2 x}$$

$$= 2 \tan x$$

$$\cdot \frac{d}{dx} [\arcsin(\frac{x-1}{x+1})]$$

$$= \frac{1}{\sqrt{1-(\frac{x-1}{x+1})^2}} \frac{x+1-(x-1)}{(x+1)^2}$$

$$= \frac{2}{(x+1)^2 \sqrt{1-(\frac{x-1}{x+1})^2}}$$

$$\cdot \frac{d}{dx} [\arctan(\arcsin(\sqrt{x}))]$$

$$= \frac{1}{1+(\arcsin(\sqrt{x}))^2}$$

$$\cdot \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{1+(\arcsin(\sqrt{x}))^2} \left( \frac{1}{2\sqrt{x}\sqrt{1-x}} \right)$$

$$\cdot \frac{d}{dx} [e^{-5x} \cos(3x)]$$

$$= -5e^{-5x} \cos(3x) - 3e^{-5x} \sin(3x)$$

$$= -e^{-5x} [5 \cos(3x) + 3 \sin(3x)]$$

$$\cdot \frac{d}{d\theta} [e^{\sin(5\theta)}]$$

$$= 5 \cos(5\theta) e^{\sin(5\theta)}$$

$$\cdot \frac{d}{dx} [e^{x \cos x}]$$

$$= (\cos x - x \sin x) e^{x \cos x}$$

$$\cdot \frac{d}{dx} [\tan(e^{3x-2})]$$

$$= 3 e^{3x-2} \sec^2(e^{3x-2})$$

$$\cdot \frac{d}{dx} [\sec(e^{\tan(x^2)})]$$

$$= \sec(e^{\tan(x^2)}) \tan(e^{\tan(x^2)})$$

$$\cdot e^{\tan(x^2)} \sec^2(x^2) 2x$$

$$= 2x \sec^2(x^2) e^{\tan(x^2)}$$

$$\cdot \sec(e^{\tan(x^2)}) \tan(e^{\tan(x^2)})$$

$$\cdot \frac{d}{dx} [\cos(\ln x)]$$

$$= -\sin(\ln x) \cdot \frac{1}{x}$$

$$= -\frac{\sin(\ln x)}{x}$$

$$\cdot \frac{d}{dy} [\ln(y^3 \sin y)]$$

$$= \frac{3y^2 \sin y + y^3 \cos y}{y^3 \sin y}$$

$$= \frac{3}{y} + \cot y$$

$$\cdot \frac{d}{dx} [(\ln(\sin x))^3]$$

$$= 3(\ln(\sin x))^2 \frac{\cos x}{\sin x}$$

$$= 3 \cot x (\ln(\sin x))^2$$

$$\cdot \frac{d}{dx} [\arcsin(2x-1)]$$

$$= \frac{2}{\sqrt{1-(2x-1)^2}}$$

$$\cdot \frac{d}{dx} [\arctan(x^3)]$$

$$= \frac{3x^2}{1+x^6}$$

$$\cdot \frac{d}{dx} [(\arcsin x)^2]$$

$$= \frac{2 \arcsin x}{\sqrt{1-x^2}}$$

$$\cdot \frac{d}{dx} [\arcsin(x^2)]$$

$$= \frac{2x}{\sqrt{1-x^4}}$$

$$\cdot \frac{d}{dx} [\arcsin x \ln x]$$

$$= \frac{\ln x}{\sqrt{1-x^2}} + \frac{1}{x} \arcsin x$$

$$\cdot \frac{d}{dx} [(1+x^2) \arctan x]$$

$$= 2x \arctan x + 1$$

$$\begin{aligned} & \frac{d}{dt} \left[ \frac{\arccos t}{t} \right] \\ &= \frac{d}{dt} [t^{-1} \arccos t] \\ &= \frac{-\frac{1}{t^2} \arccos t + \frac{-1}{t\sqrt{1-t^2}}}{t^2} \end{aligned}$$

$$\begin{aligned} & \frac{d}{dt} \left[ \arcsin \left( \frac{4}{t} \right) \right] \\ &= \frac{-\frac{4}{t^2}}{\sqrt{1 - \frac{16}{t^2}}} \end{aligned}$$

$$\begin{aligned} & \frac{d}{dt} [\sqrt{1-t^2} + \arcsin t] \\ &= \frac{-t}{\sqrt{1-t^2}} + \frac{1}{\sqrt{1-t^2}} \\ &= \frac{1-t}{\sqrt{1-t^2}} \end{aligned}$$

$$\begin{aligned} & \frac{d}{dt} [\arccos(\sqrt{2t-1})] \\ &= -\frac{1}{\sqrt{1-(2t-1)}} \cdot \frac{1}{2} (2t-1)^{-\frac{1}{2}} \cdot 2 \\ &= -\frac{1}{\sqrt{2t-1} \sqrt{2-2t}} \end{aligned}$$

$$\begin{aligned} & \frac{d}{dx} \left[ \arccos \left( \frac{b+a \cos x}{a+b \cos x} \right) \right] \\ &= -\left[ 1 - \left( \frac{b+a \cos x}{a+b \cos x} \right)^2 \right]^{-\frac{1}{2}} \\ & \quad \cdot \frac{-a \sin x (a+b \cos x) + b \sin x (b+a \cos x)}{(a+b \cos x)^2} \\ &= -\frac{1}{\sqrt{1 - \left( \frac{b+a \cos x}{a+b \cos x} \right)^2}} \cdot \frac{(b^2 - a^2) \sin x}{(a+b \cos x)^2} \end{aligned}$$

$$\begin{aligned} & \frac{d}{dx} \left[ \arctan \left( \frac{x}{a} \right) + \ln \left( \sqrt{\frac{x-a}{x+a}} \right) \right] \\ &= \frac{\frac{1}{a}}{1 + \frac{x^2}{a^2}} + \frac{1}{\sqrt{\frac{x-a}{x+a}}} \cdot \frac{1}{2} \left( \frac{x-a}{x+a} \right)^{-\frac{1}{2}} \cdot \frac{x+a - (x-a)}{(x+a)^2} \\ &= \frac{a}{x^2 + a^2} + \frac{1}{2} \cdot \frac{1}{\frac{x-a}{x+a}} \cdot \frac{2a}{(x+a)^2} \\ &= \frac{a}{x^2 + a^2} + \frac{a}{(x-a)(x+a)} \end{aligned}$$

$$\begin{aligned} & \frac{d}{dx} [\arccos(\arcsin x)] \\ &= -\frac{1}{\sqrt{1-(\arcsin x)^2}} \cdot \frac{1}{\sqrt{1-x^2}} \end{aligned}$$

$$\begin{aligned} & \frac{d}{dx} \left[ \frac{\sin^2 x}{\cos x} \right] \\ &= \frac{d}{dx} [\tan x \sin x] \\ &= \sec^2 x \sin x + \tan x \cos x \\ &= \underline{\underline{\sin x (\sec^2 x + 1)}} \end{aligned}$$

$$\begin{aligned} 2) & y = \ln(\sec x + \tan x) \\ y' &= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} \\ &= \frac{\sec x (\tan x + \sec x)}{\sec x + \tan x} \\ &= \underline{\underline{\sec x}} \\ y'' &= \underline{\underline{\sec x \tan x}} \end{aligned}$$

$$\begin{aligned} 3) & f(x) = g(\tan(\sqrt{x})) \\ f'(x) &= g'(\tan(\sqrt{x})) \\ & \quad \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} \\ &= \underline{\underline{g'(\tan(\sqrt{x})) \sec^2(\sqrt{x})}} \\ & \quad \underline{\underline{2\sqrt{x}}} \end{aligned}$$

$$\begin{aligned} 4) (a) & y = \sin x + \cos(2x), \left( \frac{\pi}{6}, 1 \right) \\ y' &= \cos x - 2\sin(2x) \\ y' \left( \frac{\pi}{6} \right) &= \cos \left( \frac{\pi}{6} \right) - 2\sin \left( \frac{\pi}{3} \right) \\ &= \frac{\sqrt{3}}{2} - 2 \left( \frac{\sqrt{3}}{2} \right) \\ &= -\frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} y &= -\frac{\sqrt{3}}{2} x + b \\ 1 &= -\frac{\pi\sqrt{3}}{12} + b \\ 1 + \frac{\pi\sqrt{3}}{12} &= b \\ y &= \underline{\underline{-\frac{\sqrt{3}}{2} x + 1 + \frac{\pi\sqrt{3}}{12}}} \end{aligned}$$

$$\begin{aligned} (b) & y = e^{-x} \sin x, (\pi, 0) \\ y' &= -e^{-x} \sin x + e^{-x} \cos x \\ &= e^{-x} (\cos x - \sin x) \\ y'(\pi) &= e^{-\pi} (\cos \pi - \sin \pi) \\ &= -e^{-\pi} \\ y &= -e^{-\pi} x + b \\ 0 &= -\pi e^{-\pi} + b \\ y &= \underline{\underline{-e^{-\pi} x + \pi e^{-\pi}}} \end{aligned}$$

$$\begin{aligned} (c) & y = 2\sin x, \left( \frac{\pi}{6}, 1 \right) \\ y' &= 2\cos x \\ y' \left( \frac{\pi}{6} \right) &= 2\cos \left( \frac{\pi}{6} \right) \\ &= 2 \left( \frac{\sqrt{3}}{2} \right) = \sqrt{3} \end{aligned}$$

$$\begin{aligned} y &= \sqrt{3} x + b \\ 1 &= \frac{\pi\sqrt{3}}{6} + b \\ 1 - \frac{\pi\sqrt{3}}{6} &= b \\ y &= \underline{\underline{\sqrt{3} x + 1 - \frac{\pi\sqrt{3}}{6}}} \end{aligned}$$

$$\begin{aligned} (d) & y = \tan x, \left( \frac{\pi}{4}, 1 \right) \\ y' &= \sec^2 x \\ y' \left( \frac{\pi}{4} \right) &= \left( \sec \left( \frac{\pi}{4} \right) \right)^2 \\ &= 2 \\ y &= 2x + b \\ 1 &= \frac{\pi}{2} + b \\ y &= \underline{\underline{2x + 1 - \frac{\pi}{2}}} \end{aligned}$$

$$\begin{aligned} (e) & y = \sec x - 2\cos x, \left( \frac{\pi}{3}, 1 \right) \\ y' &= \sec x \tan x + 2\sin x \\ y' \left( \frac{\pi}{3} \right) &= \sec \left( \frac{\pi}{3} \right) \tan \left( \frac{\pi}{3} \right) \\ & \quad + 2\sin \left( \frac{\pi}{3} \right) \\ &= 2 \left( \frac{\sqrt{3}}{1} \right) + 2 \left( \frac{\sqrt{3}}{2} \right) \\ &= 3\sqrt{3} \\ y &= 3\sqrt{3} x + b \\ 1 &= \pi\sqrt{3} + b \\ y &= \underline{\underline{3\sqrt{3} x + 1 - \pi\sqrt{3}}} \end{aligned}$$

$$5) F = \frac{\mu W}{\mu \sin \theta + \cos \theta}$$

$$\frac{dF}{d\theta} = -\mu W (\mu \sin \theta + \cos \theta)^{-2} (\mu \cos \theta - \sin \theta)$$

$$= \frac{\mu W (\sin \theta - \mu \cos \theta)}{(\mu \sin \theta + \cos \theta)^2}$$

$$6) (a) y = \sin x + \cos x, 0 \leq x \leq 2\pi$$

$$y' = \cos x - \sin x$$

$$y' = 0 \text{ for } \cos x = \sin x$$

$$\text{or for } x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\left(\frac{\pi}{4}, \sqrt{2}\right), \left(\frac{5\pi}{4}, -\sqrt{2}\right)$$

$$(b) f(x) = 2 \sin x + \sin^2 x$$

$$f'(x) = 2 \cos x + 2 \sin x \cos x$$

$$f'(x) = 0 \text{ for}$$

$$2 \cos x + 2 \sin x \cos x = 0$$

$$\text{or } \cos x + \cos x \sin x = 0$$

$$\text{or } \cos x (1 + \sin x) = 0$$

$$\cos x = 0 \text{ for } x = \frac{\pi}{2} + k\pi, \text{ for}$$

$k$  an integer

$$1 + \sin x = 0 \text{ for } \sin x = -1$$

$$\text{or for } x = \frac{3\pi}{2} + 2k\pi, \text{ for } k$$

an integer

$f(x)$  has horizontal tangent lines at the points

$$\left(\frac{\pi}{2} + 2k\pi, 2\right) \text{ and}$$

$$\left(\frac{3\pi}{2} + 2k\pi, 0\right), \text{ where } k \text{ is}$$

an integer.

$$(c) y = \sin(2x) - 2 \sin x$$

$$y' = 2 \cos(2x) - 2 \cos x$$

$$y' = 0 \text{ for } 2 \cos(2x) = 2 \cos x$$

$$\text{or for } \cos(2x) = \cos x$$

$$\text{or for } x = 2k\pi, \text{ where } \pi$$

is an integer.

$$\text{Also for } x = \pm \frac{2\pi}{3} + 2k\pi, \text{ where}$$

$k$  and  $x$  where  $k$  is an integer.

The curve  $y = \sin(2x) - 2 \sin x$  has horizontal tangent lines at the points

$$(2k\pi, 0), \text{ and}$$

$$\left(\frac{2\pi}{3} + 2k\pi, -\frac{\sqrt{3}}{2} - \sqrt{3}\right)$$

$$= \left(\frac{2\pi}{3} + 2k\pi - \frac{3\sqrt{3}}{2}\right),$$

$$\left(\frac{4\pi}{3} + 2k\pi, \frac{3\sqrt{3}}{2}\right), \text{ where } k \text{ is an integer.}$$

$$(d) f(x) = x + 2 \sin x$$

$$f'(x) = 1 + 2 \cos x$$

$$f'(x) = 0 \text{ for } \cos x = -\frac{1}{2}$$

or for  $x = \frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi$ , where  $k$  is an integer. The curve  $f(x) = x + 2 \sin x$  has horizontal tangent lines at the points

$$\left(\frac{2\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi + \sqrt{3}\right),$$

$$\left(\frac{4\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi - \sqrt{3}\right), \text{ where } k \text{ is}$$

an integer.

$$(e) y = \frac{\cos x}{2 + \sin x}$$

$$y' = \frac{-\sin x (2 + \sin x) - \cos^2 x}{(2 + \sin x)^2}$$

$$= \frac{-2 \sin x - \sin^2 x - \cos^2 x}{(2 + \sin x)^2}$$

$$= \frac{2 \sin x + \cos^2 x + \sin^2 x}{(2 + \sin x)^2}$$

$$= \frac{2 \sin x + 1}{(2 + \sin x)^2}$$

$$y' = 0 \text{ for } \sin x = -\frac{1}{2} \text{ or for}$$

$$x = \frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi, \text{ where } k$$

is an integer. The curve

$y = \frac{\cos x}{2 + \sin x}$  has horizontal tangent lines at the points:

$$\left(\frac{7\pi}{6} + 2k\pi, \frac{-\sqrt{3}}{2 - \frac{1}{2}}\right)$$

$$= \left(\frac{7\pi}{6} + 2k\pi, -\frac{\sqrt{3}}{3}\right), \text{ and}$$

$$\left(\frac{11\pi}{6} + 2k\pi, \frac{\sqrt{3}}{2 - \frac{1}{2}}\right)$$

$$= \left(\frac{11\pi}{6} + 2k\pi, \frac{\sqrt{3}}{3}\right)$$

$$7) s(t) = 10 + \frac{1}{4} \sin(10\pi t)$$

$$v(t) = s'(t) = \frac{10\pi}{4} \cos(10\pi t)$$

$$= \frac{5\pi}{2} \cos(10\pi t) \text{ cm/s.}$$

$$8) s(t) = A \cos(\omega t + \delta)$$

$$(a) v(t) = s'(t) = -A\omega \sin(\omega t + \delta)$$

$$(b) v(t) = 0 \text{ when } -A\omega \sin(\omega t + \delta) = 0$$

$$\text{or } \sin(\omega t + \delta) = 0 \text{ or}$$

$$\omega t + \delta = k\pi, \text{ where } k \text{ is an integer}$$

$$\text{or } t = \frac{k\pi - \delta}{\omega}, \text{ where } k \text{ is an integer.}$$

9)  $y(t) = A \sin(\omega t)$

$v(t) = y'(t) = A\omega \cos(\omega t)$

$a(t) = v'(t) = -A\omega^2 \sin(\omega t)$

10)  $h(x) = f(g(\sin(4x)))$

$h'(x) = f'(g(\sin(4x)))g'(\sin(4x))4\cos(4x)$

11) (a)  $f(\theta) = \sin^2 \theta, 0 \leq \theta \leq 2\pi$

$f'(\theta) = 2\sin\theta \cos\theta = 2\sin(2\theta)$

$f'(\theta) = 0$  for  $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$f''(\theta) = 2\cos(2\theta)$

$f''(\theta) = 0$  for  $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\theta$ -value	$f'$	$f''$	$f$
0		+	local min.
$0 < \theta < \frac{\pi}{4}$	+	+	incr., concave ↑
$\frac{\pi}{4}$			Pol
$\frac{\pi}{4} < \theta < \frac{\pi}{2}$	+	-	incr., concave ↓
$\frac{\pi}{2}$		-	local max.
$\frac{\pi}{2} < \theta < \frac{3\pi}{4}$	-	-	decr., concave ↓
$\frac{3\pi}{4}$			Pol
$\frac{3\pi}{4} < \theta < \pi$	-	+	decr., concave ↑
$\pi$		+	local min.
$\pi < \theta < \frac{5\pi}{4}$	+	+	incr., concave ↑
$\frac{5\pi}{4}$			Pol
$\frac{5\pi}{4} < \theta < \frac{3\pi}{2}$	+	-	incr., concave ↓
$\frac{3\pi}{2}$		-	local max.
$\frac{3\pi}{2} < \theta < \frac{7\pi}{4}$	-	-	decr., concave ↓
$\frac{7\pi}{4}$			Pol
$\frac{7\pi}{4} < \theta < 2\pi$	-	+	decr., concave ↑
$2\pi$		+	local min.

$f(0) = 0$

$f(\frac{\pi}{2}) = 1$

$f(\pi) = 0$

$f(\frac{3\pi}{2}) = 1$

$f(2\pi) = 0$

(b)  $f(t) = t + \cos t, 0 \leq t \leq 2\pi$

$f'(t) = 1 - \sin t$

$f'(t) = 0$  for  $t = \frac{\pi}{2}$

$f''(t) = -\cos t$

$f''(t) = 0$  for  $t = \frac{\pi}{2}, \frac{3\pi}{2}$

$t$ -value	$f'$	$f''$	$f$
0			local min.
$0 < t < \frac{\pi}{2}$	+	-	incr., concave ↓
$\frac{\pi}{2}$			Pol
$\frac{\pi}{2} < t < \frac{3\pi}{2}$	+	+	incr., concave ↑
$\frac{3\pi}{2}$			Pol
$\frac{3\pi}{2} < t < 2\pi$	+	-	incr., concave ↓
$2\pi$			local max.

$f(0) = 1$

$f(2\pi) = 1$