1. In this problem we will play around with exponential functions (such as $f(x) = a^x$). Pay special attention to them as they will come up later.

(a.) Find the slope of the secant line of the graph of $g(x) = 2^x$ through the points $(0, g(0))$ and $(1, g(1))$. You may use a calculator.

(b.) Repeat part a with the pairs of points $\{(0, g(0))$ and $(.1, g(.1))\}$, $\{(0, g(0))$ and $(.01, g(.01))\}$, and $\{(0, g(0))$ and $(.0001, g(.0001))\}$. Again you may use a calculator.

(c.) What do you think the slope of the tangent line of $g(x)$ at the point $x = 0$ is? What is $\ln(2)$?

(d.) Repeat parts a and b with the function $h(x) = 3^x$ instead of the function $g(x) = 2^x$. What do you think the slope of the tangent line of $h(x)$ at the point $x = 0$ is? What is $\ln(3)$?

(e.) Repeat parts a and b with the function $k(x) = e^x$. Use the button on your calculator for the number e (NOT an approximation for e you may have learned). If you wish to use google's built in calculator to calculate powers of e, for example e^2, simply enter:

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e^2
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(f.) What do you think the slope of the tangent line to e at $x = 0$ is? What is $\ln(e)$?

(g.) Do you think there are any other numbers a such that slope of the tangent line of $m(x) = a^x$ at $x = 0$ is $\ln(e)$? Speak from your gut.

2. In this problem you will investigate continuity and tangent lines further. Consider the following function:

$$f(x) = \begin{cases} x - 1, & \text{if } x \leq 0 \\ x + 1, & \text{if } x > 0 \end{cases}$$

(a.) Graph the function f.

(b.) Calculate $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to 0^-} f(x)$. What is $\lim_{x\to 0} f(x)$, and why?

(c.) Now let's try calculating the slope of a tangent line to this function. For example find the slope of the tangent line to f at $x = 1$ and $x = -1$. Remember to find the slope of the tangent line to f at $x = -1$ you need to calculate the limit

$$\lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h}$$

The limit you need to calculate to find the slope of the tangent line to f at $x = 1$ is similar.

(d.) If $a < 0$ what is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}?$$

(e.) If $a > 0$ what is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}?$$

(f.) Explain why you know that your answers in parts d and e are correct. Use graphical evidence (from the graph you drew in part a) to support your argument.

(g.) Calculate

$$\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h}$$

Note: This is LEFT limit.

(h.) Take a shot at calculating

$$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$$

Note: This is a RIGHT limit. Also note that $f(0) = -1$.

(i.) Why do you think the right limit in part e is what it is? Again use graphical evidence (from the graph you drew in part a) to support your argument.

(j.) What do you think the continuity the function f at the point $x = a$ has to do with evaluating the limit

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}?$$

(k.) Consider the function

$$w(x) = \begin{cases} x, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$$

Graph it quickly. Can you evaluate:

$$\lim_{h\to 0^-} \frac{w(0+h)-w(0)}{h}$$

or

$$\lim_{h\to 0^+} \frac{w(0+h)-w(0)}{h}?$$

What goes wrong? Explain this graphically.Note carefully that $w(0) = 1$!

(l.) Consider the function $v(x) = x$. Using what you've learned above

$$\lim_{h\to 0} \frac{v(0+h)-v(0)}{h}?$$

What goes right as compared to the function w? Explain graphically.

(m.) What property do you think a function $q(x)$ needs to have at the point $x = 0$ in order to have a chance at being able to evaluate the limit

$$\lim_{h\to 0} \frac{q(0+h)-q(0)}{h}?$$

(n.) What property do you think a function $q(x)$ needs to have at the point $x = a$ in order to have a chance at being able to evaluate the limit

$$\lim_{h\to 0} \frac{q(a+h)-q(a)}{h}?$$

(o.) Do you think that this property is enough to ensure that the limit

$$\lim_{h\to 0} \frac{q(a+h)-q(a)}{h}$$

exists? Examine the function

$$p(x) = \begin{cases} -x, & \text{if } x \leq 0 \\ x, & \text{if } x > 0 \end{cases}$$

at the point $x = 0$ function before you say anything too bold. This function is often called the absolute value function, and is often written $p(x) = |x|$ for short. It was featured on the first midterm exam.