

Midterm Exam 2
Math 1
November 9, 2011

Name: _____

Please circle your instructor's name below:

Harris

LaLonde

Please read the following instructions before starting the exam:

- This exam is closed-book, with no calculators, notes, or books allowed. You may not give or receive any help on the exam, though you may ask the instructors for clarification if necessary.
- Be sure to show all your work wherever possible. Even if your final answer is incorrect, we can assign an appropriate amount of partial credit if we can see how you arrived at your answer.
- Please circle or otherwise indicate your final answer.
- This test has a total of 9 questions, worth a total of 100 points. Point values are indicated for each question.
- You will have two hours from the start of the exam to complete it.
- Good luck!

HONOR STATEMENT: I have neither given nor received any help on this exam, and I attest that all of the answers are my own work.

SIGNATURE: _____

This page is for grading purposes only.

Problem	Score	Points
1		10
2		8
3		12
4		15
5		10
6		10
7		15
8		12
9		8
Total		100

1. [10 points] **Multiple choice.** Circle the correct answer for each question. Each part is worth 2 points.

(a) Suppose that $f(x)$ is a function satisfying $\lim_{x \rightarrow 2^-} f(x) = 4$, $\lim_{x \rightarrow 2^+} f(x) = 0$, and $f(2) = 0$. Then f has a _____ at $x = 2$.

- A. removable discontinuity
- B. jump discontinuity
- C. infinite discontinuity
- D. corner

(b) Suppose that when x is near 1,

$$x^4 - 4x^3 + 6x^2 - 4x + 2 \leq f(x) \leq 2x^2 - 4x + 3.$$

What is $\lim_{x \rightarrow 1} f(x)$?

- A. 1
- B. 0
- C. 3
- D. -1

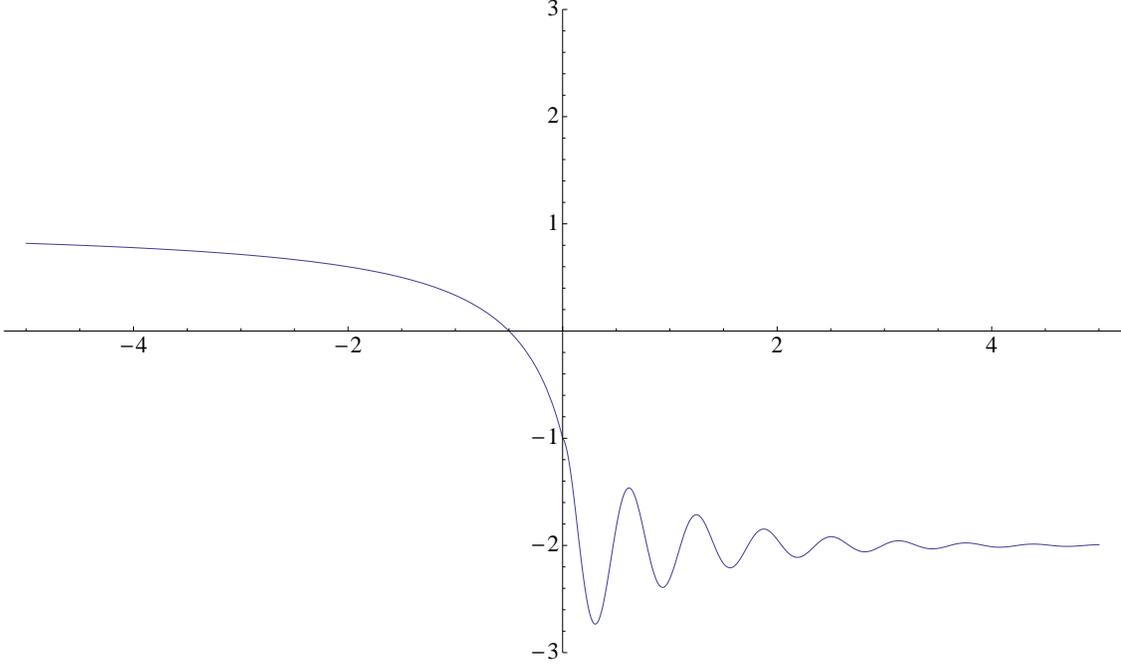
(c) If $f(x)$ and $g(x)$ are differentiable functions, what is the formula for the derivative of $fg(x)$?

- A. $f'(x)g'(x)$
- B. $f'(x)g'(x) + f(x)g(x)$
- C. $f'(x)g(x) + f(x)g'(x)$
- D. $f'(x)g(x)$

(d) $\lim_{x \rightarrow \infty} \sin(x) = ?$

- A. 1
- B. $+\infty$
- C. $-\infty$
- D. The limit does not exist.

(e) If f has the following graph, what are the equations of the horizontal asymptotes?



- A. $x = 0$ and $x = 1$
- B. $x = 1$ and $x = -2$
- C. $y = 0$ and $y = -1$
- D. $y = 1$ and $y = -2$

2. [8 points] **Short answer.** Answer each of the following questions as precisely as possible. Each part is worth 2 points.

(a) Complete the following statement:

A function $f(x)$ is continuous at a real number a if . . .

(b) State the definition of the derivative of a function $f(x)$ at a real number a .

(c) Complete the statement of the following theorem:

“If $f(x)$ is continuous on an interval $[a, b]$ and N is a number satisfying $f(a) < N < f(b)$, then . . .”

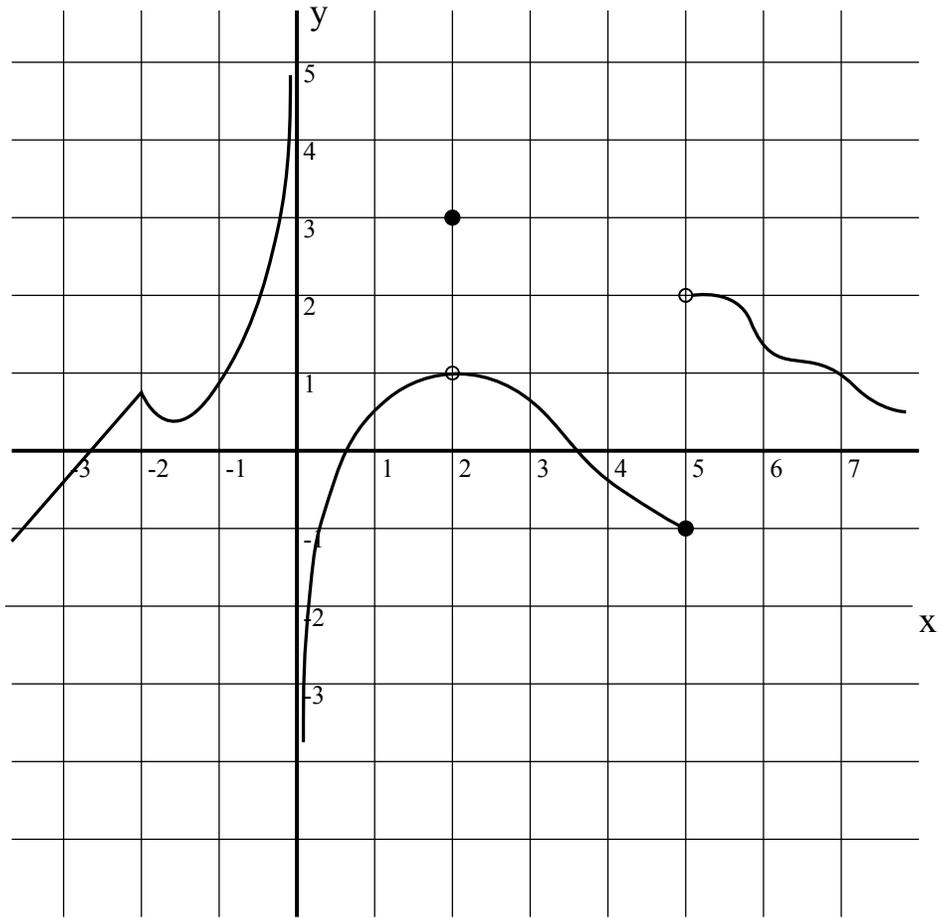
What is this theorem called?

(d) Complete the statement of the following theorem:

“If $g(x) \leq f(x) \leq h(x)$ and $\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x)$, then . . .”

What is this theorem called?

3. [12 points] Below is the graph of a function $f(x)$.



(a) [2 points] What are $\lim_{x \rightarrow 5^-} f(x)$ and $\lim_{x \rightarrow 5^+} f(x)$?

(b) [2 points] What are $\lim_{x \rightarrow 0^-} f(x)$ and $\lim_{x \rightarrow 0^+} f(x)$?

(c) [2 points] What are $\lim_{x \rightarrow 2} f(x)$ and $f(2)$?

(d) [3 points] At what numbers is f discontinuous?

(e) [3 points] State all the numbers at which $f(x)$ is **not** differentiable.

4. [15 points] Compute each of the following limits.

(a) [5 points]

$$\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x - 1}$$

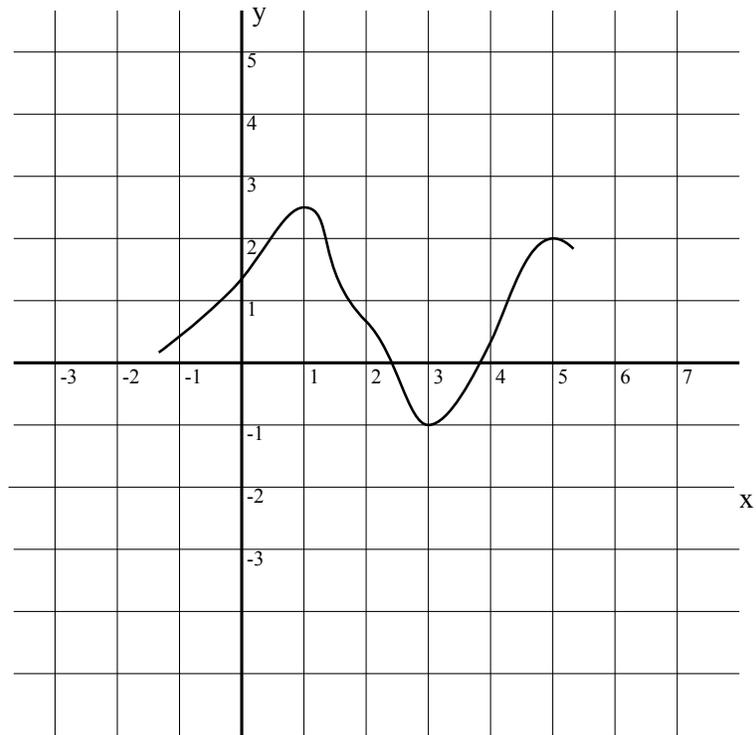
(b) [5 points]

$$\lim_{u \rightarrow 0} \frac{\sqrt{u+1} - 1}{u}$$

(c) [5 points]

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 4x + 7}{x^2 + x + 1}$$

5. [10 points] Below is the graph of a function $f(x)$.



(a) [6 points] Estimate the values of:

(i) $f'(0)$

(ii) $f'(1)$

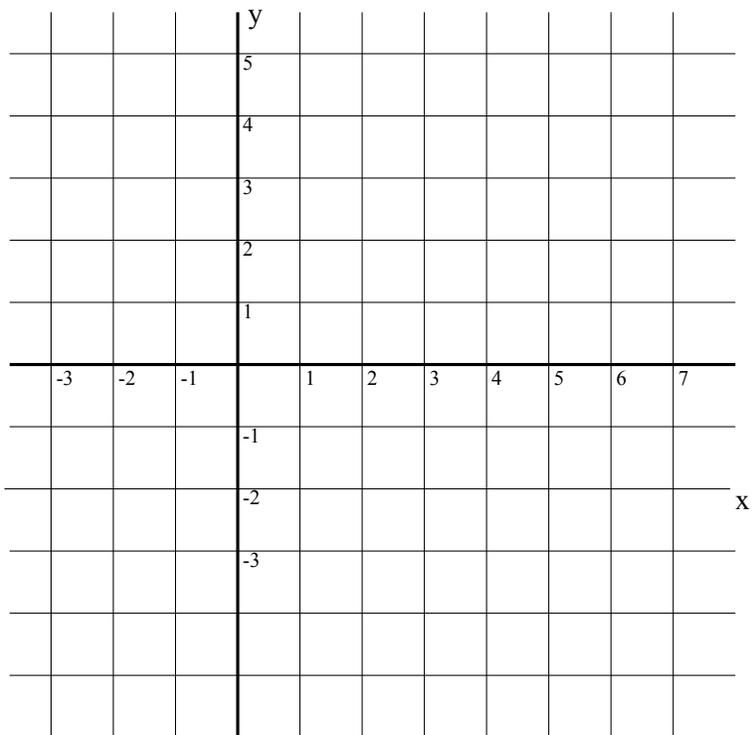
(iii) $f'(2)$

(iv) $f'(3)$

(v) $f'(4)$

(vi) $f'(5)$

(b) [4 points] Sketch the graph of $f'(x)$.



6. [10 points] Let

$$f(x) = \sqrt{4x + 1}.$$

(a) [8 points] Find $f'(x)$ **from the definition of the derivative**.

(b) [2 points] State where $f(x)$ is differentiable.

7. [15 points] Find the derivative of each of the following functions. (Simplify wherever possible.)

(a) [5 points]

$$f(x) = e^x + 7x^2 - 2$$

(b) [5 points]

$$g(x) = \sqrt{x}(e^x + 1)$$

(c) [5 points]

$$h(x) = \frac{x^4 - 2x}{x + 1}$$

8. [12 points] Let $g(x) = 5x^2 - 10x + \frac{4}{x}$.

(a) [5 points] Find the derivative $g'(x)$.

(b) [2 points] Find $g'(2)$.

(c) [5 points] Find the equation of the tangent line to the graph of $g(x)$ at the point where $x = 2$.

9. [8 points] The height of a flying fish follows the equation $h(t) = -t^2 + 8t - 12$ meters, where t is measured in seconds and $h(t) = 0$ corresponds to the surface of the water.

(a) [3 points] At what time(s) is the flying fish at sea level?

(b) [5 points] Find the instantaneous velocity of the fish at each of these times. Include proper units.