

Final Exam  
Math 1  
November 16, 2012

Name: \_\_\_\_\_

Please circle your instructor's name below:

Harnish

Zhao

**Please read the following instructions before starting the exam:**

- This exam is closed-book, with no calculators, notes, or books allowed. You may not give or receive any help on the exam, though you may ask the instructors for clarification if necessary.
- Be sure to show all your work wherever possible. Even if your final answer is incorrect, we can assign an appropriate amount of partial credit if we can see how you arrived at your answer.
- Please circle or otherwise indicate your final answer.
- This test has a total of 11 questions, worth a total of 150 points. Point values are indicated for each question.
- You will have three hours from the start of the exam to complete it.
- Good luck!

HONOR STATEMENT: I have neither given nor received any help on this exam, and I attest that all of the answers are my own work.

SIGNATURE: \_\_\_\_\_

This page is for grading purposes only.

Problem	Points	Score
1	12	
2	21	
3	10	
4	13	
5	10	
6	16	
7	20	
8	8	
9	5	
10	20	
11	15	
Total	150	

# Geometric Formulas

Area of a Rectangle:  $A = bh$

Area of a Square with side  $s$ :  $A = s^2$

Area of a Triangle:  $A = \frac{1}{2}bh$

Pythagorean Theorem:  $a^2 + b^2 = c^2$

Area of a Circle:  $A = \pi r^2$

Circumference of a Circle:  $C = 2\pi r$

Volume of a Sphere:  $V = \frac{4}{3}\pi r^3$

Surface Area of a Sphere:  $A = 4\pi r^2$

Volume of a Cylinder:  $V = \pi r^2 h$

Volume of a Cone:  $V = \frac{1}{3}\pi r^2 h$

Surface Area of a Cone:  $A = \pi r\sqrt{r^2 + h^2}$

Volume of a Rectangular Prism (a box with a rectangular base):  $V = lwh$

Surface Area of a Rectangular Prism:  $A = 2lw + 2lh + 2wh$

# Trigonometric Values

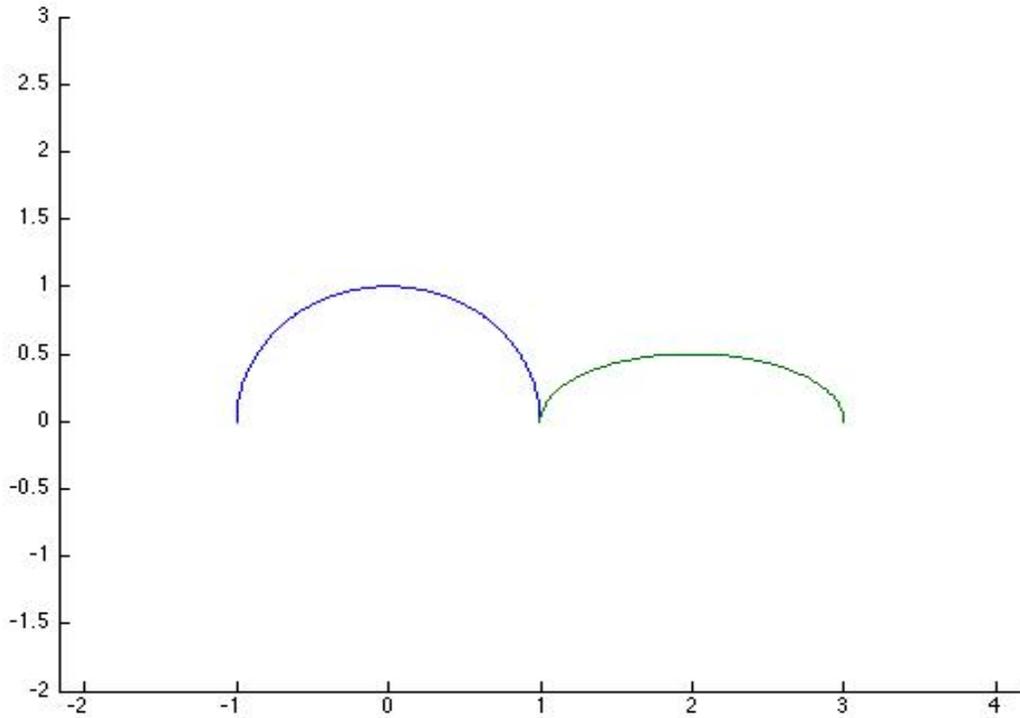
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(x)$	0	$\frac{\sqrt{3}}{2}$	1	$\sqrt{3}$	$\infty$

1. [12 pts] **Multiple choice.** Circle the correct answer for each question. Each part is worth 3 points.

(a) What is the domain of  $f(x) = \frac{x^2+3x-7}{x^2-4x-12}$  ?

- A.  $(-\infty, -2) \cup (6, \infty)$
- B.  $(-\infty, -2) \cup (-2, 6) \cup (6, \infty)$
- C.  $(-\infty, 6) \cup (6, -2) \cup (-2, \infty)$
- D.  $(-\infty, 6) \cup (-2, \infty)$

(b) Based on the graphs of  $f(x)$  and  $g(x)$ , what is  $g(x)$  in terms of  $f(x)$ ?  $f(x)$  is on the left and  $g(x)$  is on the right.



- A.  $g(x) = \frac{1}{2}f(x - 2)$
- B.  $g(x) = \frac{1}{2}f(x + 2)$
- C.  $g(x) = 2f(x + 2)$
- D.  $g(x) = 2f(x - 2)$

(c) Which of the following is the inverse function of  $y = \frac{x}{x+1}$ ?

A.  $y = \frac{x}{1+x}$

B.  $y = \frac{1}{1+x}$

C.  $y = \frac{1}{1-x}$

D.  $y = \frac{x}{1-x}$

(d) If  $f'(x) = g'(x)$  for all real numbers, what can you conclude about  $f(x)$  and  $g(x)$ ?

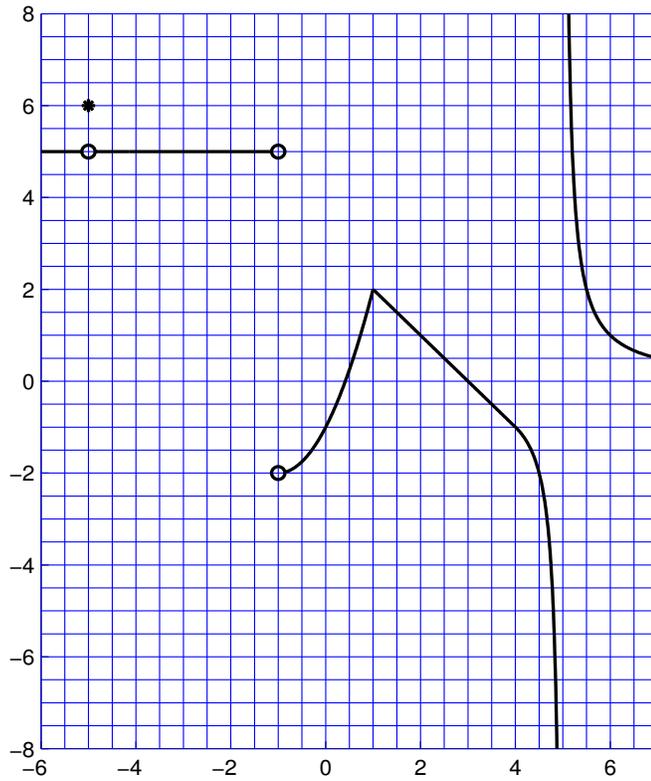
A.  $f(x) + g(x) = C$ , where  $C$  is a constant

B.  $f(x) - g(x) = C$ , where  $C$  is a constant

C.  $f(x)g(x) = C$ , where  $C$  is a constant

D.  $\frac{f(x)}{g(x)} = C$ , where  $C$  is a constant

2. [21 pts] Consider the following graph of a function  $f(x)$ :



(a) [3 pts.] State the intervals where  $f$  is continuous.

(b) [3 pts.] State each discontinuity and name what type of discontinuity it is.  
(i.e. jump discontinuity, removable discontinuity, or vertical asymptote)

(c) [3 pts.] State the intervals where  $f$  is differentiable.

(d) [2 pts.] What is  $\lim_{x \rightarrow 1^-} f(x)$ ?

(e) [2 pts.] What is  $\lim_{x \rightarrow 1^+} f(x)$ ?

(f) [2 pts.] What is  $\lim_{x \rightarrow 1} f(x)$ ?

(g) [2 pts.] What is  $\lim_{x \rightarrow -1^-} f(x)$ ?

(h) [2 pts.] What is  $\lim_{x \rightarrow -1^+} f(x)$ ?

(i) [2 pts.] What is  $\lim_{x \rightarrow -1} f(x)$ ?

3. [10 pts] Evaluate the following limits.

(a) [5 pts.]  $\lim_{x \rightarrow 0} \frac{x^2}{1 - \sin(x)}$

(b) [5 pts.]  $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos(x)}$

4. [13 pts] Derivatives.

(a) [5 pts.] Find  $\frac{dy}{dx}$  if  $y = (\ln(x) + \tan(x))^2$ .

(b) [8 pts.] Find an equation of the tangent line to the curve defined by  $y = e^{\frac{\sin(x)}{x}}$  at  $(1, e^{\sin(1)})$ .

5. [10 pts] Find  $\frac{dy}{dx}$  if  $\sin(x - y) = y^2 \sec(x)$ .

6. [16 pts] Find  $\frac{dy}{dx}$  using Logarithmic Differentiation.

(a) [8 pts.]  $y = \frac{(x-1)\sqrt{x^2+1}}{(x^3+2x+1)^2}$ .

(b) [8 pts.]  $y = (\cos(x))^x$ .

7. [20 pts]

- (a) [10 pts.] Each side of a square is increasing at a rate of  $6 \text{ cm/s}$ . At what rate is the area of the square increasing when the area of the square is  $16 \text{ cm}^2$ .

(b) [10 pts.] The radius of a sphere is increasing at a rate of  $4 \text{ mm/s}$ . How fast is the volume increasing when the radius is  $80 \text{ mm}$ .

8. [8 pts]

(a) [4 pts.] Find the critical numbers of the function  $f(x) = x^3 - 6x^2 + 5$ .

(b) [4 pts.] Find the absolute maximum and absolute minimum values of  $f$  on the interval  $[-3, 5]$ .

9. [5 pts] If  $f(1) = 10$  and  $f'(x) \geq 2$  for  $1 \leq x \leq 4$ , how small can  $f(4)$  possibly be?

10. [20 pts] Let  $f(x) = 2x^3 - 3x^2 - 36x$ .

(a) [3 pts.] Find the intervals of increase or decrease.

(b) [3 pts.] Find the local maximum and minimum values.

(c) [3 pts.] Find the intervals of concavity.

(d) [3 pts.] Find the inflection points, if any.

(e) [3 pts.] Find the y-intercept.

(f) [5 pts.] Sketch the graph of  $f$ .

**11.** [15 pts] A box with a square base and open top must have a volume of  $32,000 \text{ cm}^3$ . Find the dimensions of the box that minimize the amount of material used.