Math 1: Calculus with Algebra **Quiz 4** Name: Answer Key
Fall 2015

Instructions: (24 points) This quiz consists of 4 problems covering material from the 5th week of class. Credit is awarded for correct solutions in which you **show your work**. You will have 30 minutes to complete this quiz. You may not use a calculator, textbook, notes, or any outside source while taking this quiz.

(6^{pts}) **1.** Find the following limits (if they exist):

(a) $\lim_{x\to 2} \dfrac{x^3 - 2x - 1}{x - 3}$

Solution:

$$\lim_{x\to 2} \frac{x^3 - 2x - 1}{x - 3} = \frac{(2)^3 - 2(2) - 1}{2 - 3} = \frac{8 - 4 - 1}{-1} = -3$$

(b) $\lim_{x\to 0} \dfrac{\sqrt{x+4} - 2}{x}$

Solution:

$$\lim_{x\to 0} \frac{\sqrt{x+4} - 2}{x} \cdot \frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2} = \lim_{x\to 0} \frac{(x+4) - 2^2}{x(\sqrt{x+4} + 2)} = \lim_{x\to 0} \frac{1}{\sqrt{x+4} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{4}.$$

(6^{pts}) **2.** (Continuity)

(a) Let f be a function defined near a. State the definition of f being continuous at a.

Solution: We say f is continuous at a if $\lim_{x\to a} f(x) = f(a)$.

(b) Determine whether $g(x) = \begin{cases} \sqrt{x^2 + 2x + 1} & x < -2 \\ x + 3 & x \geq -2 \end{cases}$ is continuous at $x = -1$.

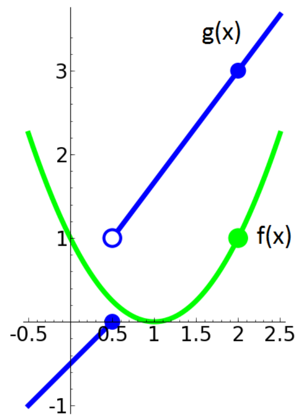
Solution:

- $g(-1) = (-1) + 3 = 2$.
- $\lim_{x\to -1^-} g(x) = \lim_{x\to -1^-} (x + 3) = (-1) + 3 = 2$.
- $\lim_{x\to -1^+} g(x) = \lim_{x\to -1^+} (x + 3) = (-1) + 3 = 2$.

These last two parts tell us that $\lim_{x\to -1} g(x) = 2$. Since $\lim_{x\to -1} g(x) = g(-1)$, we conclude that g is continuous at $x = -1$.

Alternate solution: $\lim_{x\to -1} g(x) = \lim_{x\to -1}(x + 3) = (-1) + 3 = 2$ (since around -1, g is defined by the same piece).

(6^{pts}) **3.** Determine the limits using the function graphs below:



(a) $\lim_{x\to 2} \frac{f(x)}{g(x)}$

Solution: Since $\lim_{x\to 2} f(x) = 1$ and $\lim_{x\to 2} g(x) = 3$,

$$\lim_{x\to 2} \frac{f(x)}{g(x)} = \frac{1}{3}.$$

(b) $\lim_{x\to \frac{1}{2}^+} g(x)$

Solution: As we approach from the right, we're using the second piece of g:

$$\lim_{x\to \frac{1}{2}^+} g(x) = 1.$$

(6^{pts}) **4.** Consider the function $h(x) = \begin{cases} -\frac{1}{x} & -1 < x < 0 \\ \frac{1}{x} & 0 < x < 1 \\ x^2 + 2 & 1 \le x < \frac{3}{2} \end{cases}$

(a) Find the vertical asymptote(s) of h.

Solution: $x = 0$ is a vertical asymptote since $\lim_{x\to 0^+} h(x) = \lim_{x\to 0^+} \frac{1}{x} = \infty$.

(b) Find the discontinuities of h. For each one, label the type of discontinuity.

Solution: Both $-\frac{1}{x}$ and $\frac{1}{x}$ have an essential discontinuity at $x = 0$ coming from the vertical asymptote.
$x^2 + 2$ is a polynomial so it has no discontinuities.
So the only places we need to check are $x = 0$ and the breaking points between the different pieces of h: $x = 0, 1.$[1]
$\boxed{x = 0 \text{ is an essential discontinuity}}$ because it's a vertical asymptote.
Checking $x = 1$:

$$\lim_{x\to 1^-} h(x) = \lim_{x\to 1^-} 1/x = 1$$

but

$$\lim_{x\to 1^+} h(x) \lim_{x\to 1^+} (x^2 + 2) = 3.$$

So both one-sided limits exist and are finite. Hence it is a jump discontinuity.
$\boxed{x = 1 \text{ is a jump discontinuity}}$.

[1]This means that the $x = 0$ is redundant.