

Midterm Exam 1
Math 1
October 19, 2011

Name: _____

Please circle your instructor's name below:

Harris

LaLonde

Please read the following instructions before starting the exam:

- This exam is closed-book, with no calculators, notes, or books allowed. You may not give or receive any help on the exam, though you may ask the instructors for clarification if necessary.
- Be sure to show all your work wherever possible. Even if your final answer is incorrect, we can assign an appropriate amount of partial credit if we can see how you arrived at your answer.
- Please circle or otherwise indicate your final answer.
- This test has a total of 9 questions, worth a total of 100 points. Point values are indicated for each question.
- You will have two hours from the start of the exam to complete it.
- Good luck!

HONOR STATEMENT: I have neither given nor received any help on this exam, and I attest that all of the answers are my own work.

SIGNATURE: _____

This page is for grading purposes only.

Problem	Score	Points
1		10
2		15
3		7
4		10
5		10
6		16
7		12
8		10
9		10
Total		100

This sheet is for reference.

Common trigonometric ratios:

θ	radians	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45°	$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	-

1. [10 points] **Multiple choice.** Circle the correct answer for each question. Each part is worth 2 points.

(a) To obtain the graph of the function $f(x + 2)$ from the graph of $f(x)$, you would

- A. shift the graph of f up by 2 units.
- B. shift the graph of f to the left by 2 units.
- C. shift the graph of f to the right by 2 units.
- D. stretch the graph of f vertically by a factor of 2.

(b) Suppose that a and b are positive real numbers. Which of the following is true?

- A. $\ln(a + b) = \ln(a) + \ln(b)$
- B. $\ln(ab) = \ln(a) + \ln(b)$
- C. $\ln(a + b) = \ln(a) \ln(b)$
- D. $\ln(ab) = \ln(a) \ln(b)$

(c) If $f(x) = x^2 + 1$ and $g(x) = \tan(x)$, what is $f \circ g$?

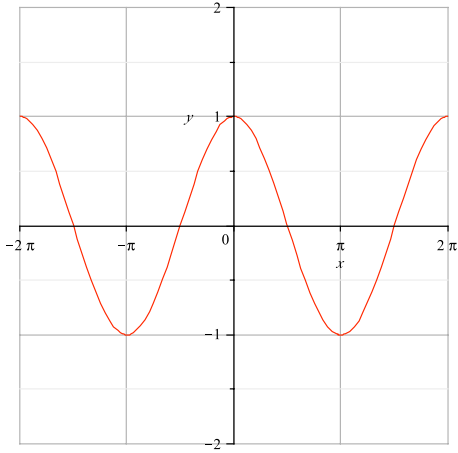
- A. $\tan(x^2 + 1)$
- B. $\tan^2(x) + 1$
- C. $\tan(x^2) + 1$
- D. $\tan^2(x) + \tan(1)$

(d) Suppose that when we draw the angle θ in standard position, its terminal side lies in the third quadrant. Which of the following is true regarding the values of the trigonometric functions at θ ?

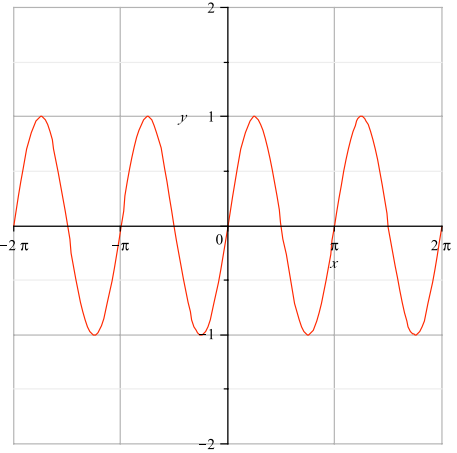
- A. $\sin(\theta) > 0$, $\cos(\theta) < 0$, and $\tan(\theta) < 0$
- B. $\sin(\theta) > 0$, $\cos(\theta) > 0$, and $\tan(\theta) < 0$
- C. $\sin(\theta) < 0$, $\cos(\theta) < 0$, and $\tan(\theta) > 0$
- D. $\sin(\theta) < 0$, $\cos(\theta) > 0$, and $\tan(\theta) < 0$

(e) Which of the following figures correctly displays the graph of $y = \cos(2x)$?

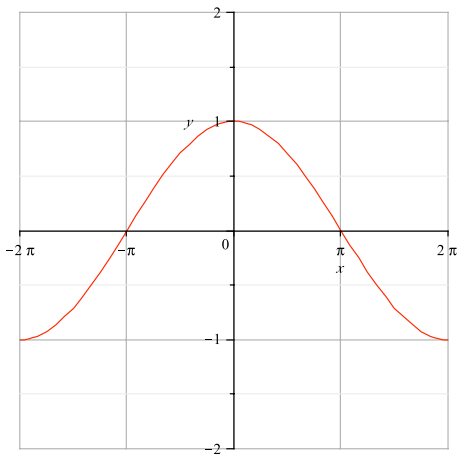
A.



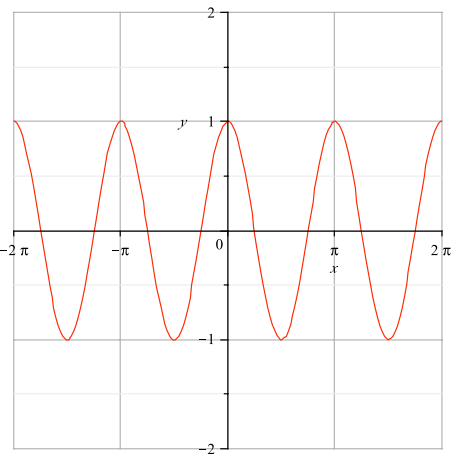
C.



B.



D.



2. [15 points] Find the domain of each of the following functions.

(a) [5 points]

$$f(x) = \frac{x + 1}{x^2 + x - 12}$$

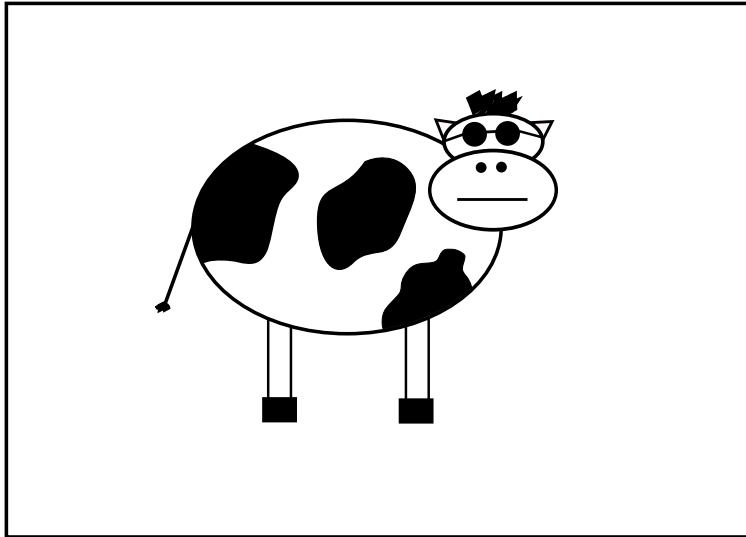
(b) [5 points]

$$g(x) = \sqrt{5 - x} + \sqrt{x + 3}$$

(c) [5 points]

$$h(x) = \sqrt[3]{3x - 1}$$

3. [7 points] You've decided to start raising cows, and you need to fence in a pasture for them to graze. The pasture is rectangular and covers an area of 250,000 square feet, and fencing costs \$7 per foot. Write the total cost of fencing in the pasture as a function of the length x of one of its sides.



4. [10 points] Let

$$f(x) = \frac{x+1}{x}$$

$$g(x) = \frac{1}{x-1}.$$

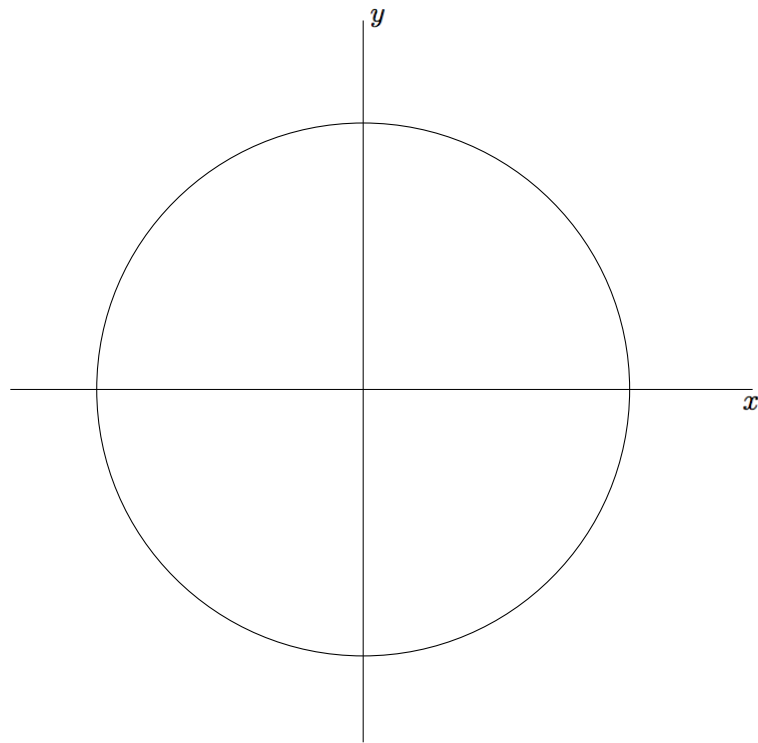
(a) [4 points] Find $(f \circ g)(x)$.

(b) [4 points] Find $(g \circ f)(x)$.

(c) [2 points] Based on what you found for $f \circ g$ and $g \circ f$, how are f and g related?

5. [10 points] Let $\theta = \frac{5\pi}{6}$.

(a) [4 points] Draw the angle θ in standard position.



(b) [6 points] Find $\sin \theta$, $\cos \theta$, and $\tan \theta$.

6. [16 points] Simplify the following expressions:

(a) [4 points]

$$\frac{a^2b^5}{ab^{1/3}}$$

(b) [4 points]

$$\left(\frac{a^{1/2}b^{1/4}}{b^{1/2}}\right)^2$$

(c) [4 points]

$$\log_5(\ln(e^5))$$

[4 points] Write the following expression as a single logarithm:

(d)

$$2\ln(a) + \ln(a - b) - \ln(c)$$

7. [12 points] Consider the function

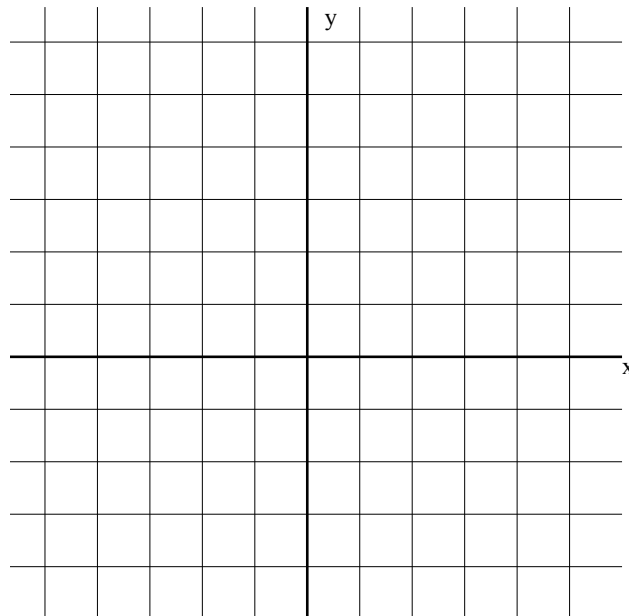
$$f(x) = 2 + e^{5x}.$$

(a) [2 points] What are the domain and range of f ?

(b) [4 points] Find the inverse function f^{-1} .

(c) [2 points] What are the domain and range of f^{-1} ?

(d) [4 points] Sketch the graphs of f and f^{-1} below. Be sure to label each curve accordingly.



8. [10 points] Suppose that the position of an object is given by the function

$$f(x) = \frac{1}{2}x^2.$$

That is, after x seconds, the object has traveled $\frac{1}{2}x^2$ feet from its starting position.

(a) [4 points] Find the average velocity of the object over the interval $[1, 2]$.

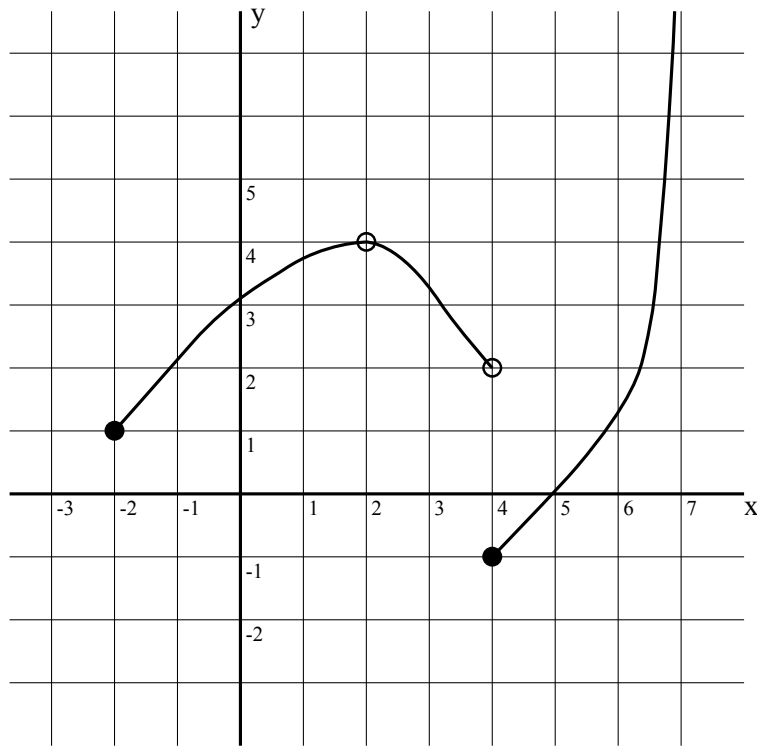
(b) [4 points] Find the average velocity of the object over the interval $[0, 1]$.

(c) [2 points] Suppose that the average velocity of an object over the interval $[1, 1 + h]$ for various values of h is given by the following table:

h	1	0.1	0.01	0.001	0.0001	0.00001
avg. velocity over $[1, 1 + h]$	2	1.12	1.0951	1.01314	1.00256	1.000423

We can say that the _____ of the object
at $x = 1$ is _____.

9. [10 points] Suppose that f is a function with the following graph:



Find each of the following quantities [2 points each]:

(a) $\lim_{x \rightarrow 2} f(x)$

(b) $\lim_{x \rightarrow 4} f(x)$

(c) $\lim_{x \rightarrow 4^-} f(x)$

(d) $\lim_{x \rightarrow 4^+} f(x)$

(e) $\lim_{x \rightarrow 7^-} f(x)$