MATH 1 LECTURE 1 MONDAY 09-12-16

MICHAEL MUSTY

Contents

I. Course Information

start 10:10am Bartlett 105

WRITE

Course Website: `https://math.dartmouth.edu/~m1f16/`

NOTE

- Surveys
- x-hours
- WebWork
- Lecture notes and where they will be posted

II. Definition of a Function

10:25am

WRITE

Def: A function is a rule that assigns to every input in the domain a unique output in the codomain.

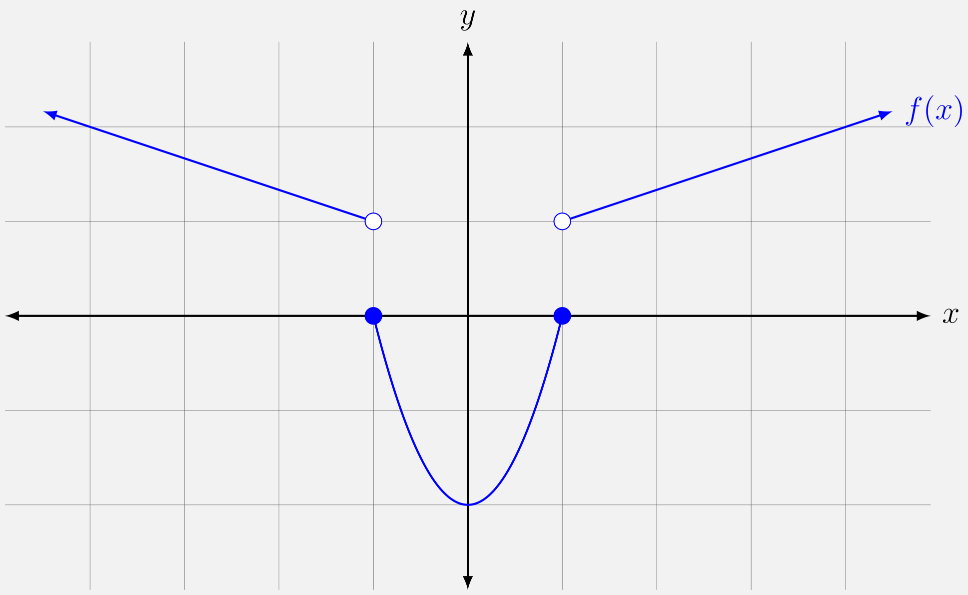
NOTE

- $f(x) = \#$ prime divisors of x.
- more examples with domain and range...
- Mention real numbers, rational numbers, natural numbers, etc.
- Notation $f : A \to B$.

- Notation $\in$ means "in".
- Notation $\cup$ means "or".
- Notation $\cap$ means "and".
- Notation $\subseteq$ means "is contained in".

WRITE

Example:



10:45am

III. Combining Functions

NOTE

- Define abstractly what it means to $+, -, *, /, \circ$ functions $f : \mathbb{R} \to \mathbb{R}$.
- Then do some examples.

WRITE

Example:
Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x + 3$.
Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = x^2 - 5$.
Then we can define new functions $f + g, f - g, f \cdot g, f/g, f \circ g, g \circ f, f \circ f, g \circ g$:

- $f + g : \mathbb{R} \to \mathbb{R}$ is the function defined by

$$\begin{aligned}(f + g)(x) &= 2x + 3 + x^2 - 5 \\ &= x^2 + 2x - 2.\end{aligned}$$

- $f - g : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(f-g)(x) &= 2x + 3 - (x^2 - 5) \\ &= -x^2 + 2x + 8.\end{aligned}$$
- $f \cdot g : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(f \cdot g)(x) &= (2x+3) \cdot (x^2 - 5) \\ &= (2x)(x^2) + (2x)(-5) + (3)(x^2) + (3)(-5) \\ &= 2x^3 - 10x + 3x^2 - 15 \\ &= 2x^3 + 3x^2 - 10x - 15.\end{aligned}$$
- $f/g : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$(f/g)(x) = \frac{2x+3}{x^2-5}.$$
Note that the domain of f/g does not include $\pm\sqrt{5} \in \mathbb{R}$.
- $f \circ g : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(f \circ g)(x) &= 2(\underbrace{x^2 - 5}_{g(x)}) + 3 \\ &= 2x^2 - 10 + 3 \\ &= 2x^2 - 7.\end{aligned}$$
- $g \circ f : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(g \circ f)(x) &= (\underbrace{2x + 3}_{f(x)})^2 - 5 \\ &= 4x^2 + 6x + 6x + 9 - 5 \\ &= 4x^2 + 12x + 4.\end{aligned}$$
- $f \circ f : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(f \circ f)(x) &= 2(2x+3) + 3 \\ &= 4x + 6 + 3 \\ &= 4x + 9.\end{aligned}$$
- $g \circ g : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(g \circ g)(x) &= (x^2 - 5)^2 - 5 \\ &= x^4 - 5x^2 - 5x^2 + 25 - 5 \\ &= x^4 - 10x^2 + 20.\end{aligned}$$

IV. Sequences

10:55am

WRITE

Def.:
A sequence of real numbers is a subset of $\mathbb{R}$ indexed by the natural numbers. We write $\{a_n\}_{n=1}^{\infty}$ to denote the sequence $a_1, a_2, a_3, \ldots$.

WRITE

Example: The sequence $\{2n+5\}_{n=1}^{\infty}$ can more descriptively be written as

$$\underbrace{2\cdot 1+5}_{n=1}, \underbrace{2\cdot 2+5}_{n=2}, \underbrace{2\cdot 3+5}_{n=3}, \cdots.$$

NOTE

- `https://oeis.org/`
- Functions can give us sequences by just plucking out some values. For example let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=2x$. Then $\{f(n)\}_{n=1}^{\infty}$ defines the sequence of positive even integers:

$$2, 4, 6, 8, \ldots.$$

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end
11:15am