MATH 1 LECTURE 2 WEDNESDAY 09-14-16

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I. Reminders/Announcements

start 10:10am Bartlett 105

NOTE

- Wednesday WebWork due Friday.
- Thursday Problem Session during x-hour.
- Written HW#1 assigned.

II. Combining Functions

10:12am

WRITE

Example:
Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x + 3$.
Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = x^2 - 5$.
Then we can define new functions $f + g, f - g, f \cdot g, f/g, f \circ g, g \circ f, f \circ f, g \circ g$:

- $f + g : \mathbb{R} \to \mathbb{R}$ is the function defined by

$$\begin{aligned}(f+g)(x) &= 2x + 3 + x^2 - 5 \\ &= x^2 + 2x - 2.\end{aligned}$$

- $f - g : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(f-g)(x) &= 2x + 3 - (x^2 - 5) \\ &= -x^2 + 2x + 8.\end{aligned}$$
- $f \cdot g : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(f \cdot g)(x) &= (2x+3) \cdot (x^2 - 5) \\ &= (2x)(x^2) + (2x)(-5) + (3)(x^2) + (3)(-5) \\ &= 2x^3 - 10x + 3x^2 - 15 \\ &= 2x^3 + 3x^2 - 10x - 15.\end{aligned}$$
- $f/g : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$(f/g)(x) = \frac{2x+3}{x^2-5}.$$
Note that the domain of f/g does not include $\pm\sqrt{5} \in \mathbb{R}$.
- $f \circ g : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(f \circ g)(x) &= 2(\underbrace{x^2 - 5}_{g(x)}) + 3 \\ &= 2x^2 - 10 + 3 \\ &= 2x^2 - 7.\end{aligned}$$
- $g \circ f : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(g \circ f)(x) &= (\underbrace{2x+3}_{f(x)})^2 - 5 \\ &= 4x^2 + 6x + 6x + 9 - 5 \\ &= 4x^2 + 12x + 4.\end{aligned}$$
- $f \circ f : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(f \circ f)(x) &= 2(2x+3) + 3 \\ &= 4x + 6 + 3 \\ &= 4x + 9.\end{aligned}$$
- $g \circ g : \mathbb{R} \to \mathbb{R}$ is the function defined by
$$\begin{aligned}(g \circ g)(x) &= (x^2 - 5)^2 - 5 \\ &= x^4 - 5x^2 - 5x^2 + 25 - 5 \\ &= x^4 - 10x^2 + 20.\end{aligned}$$

III. Sequences

10:20am

WRITE

Def.:
A sequence of real numbers is a subset of $\mathbb{R}$ indexed by the natural numbers. We write $\{a_n\}_{n=1}^{\infty}$ to denote the sequence $a_1, a_2, a_3, \ldots$.

WRITE

Example: The sequence $\{2n+5\}_{n=1}^{\infty}$ can more descriptively be written as

$$\underbrace{2\cdot 1+5}_{n=1}, \underbrace{2\cdot 2+5}_{n=2}, \underbrace{2\cdot 3+5}_{n=3}, \cdots .$$

NOTE

- `https://oeis.org/`
- Functions can give us sequences by just plucking out some values. For example let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x$. Then $\{f(n)\}_{n=1}^{\infty}$ defines the sequence of positive even integers:

$$2, 4, 6, 8, \ldots .$$

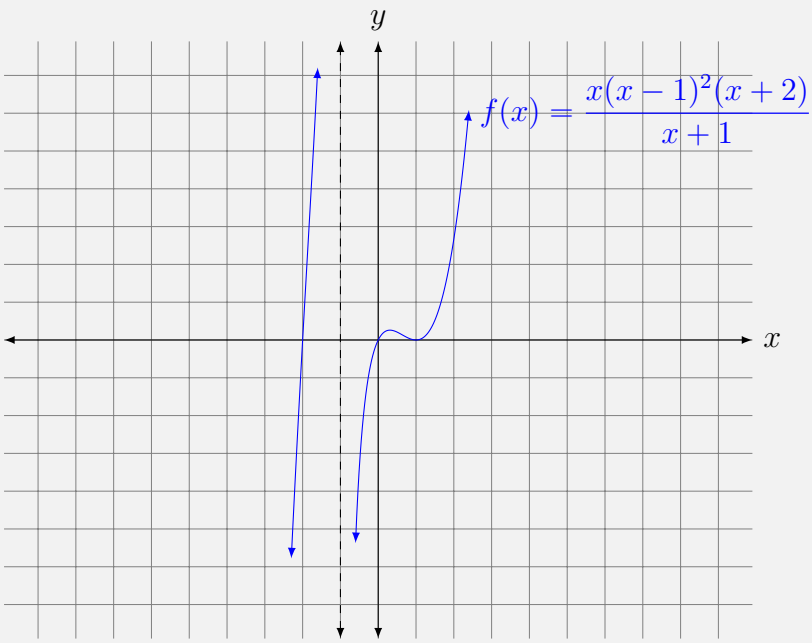
IV. Graphs of Functions

10:30am

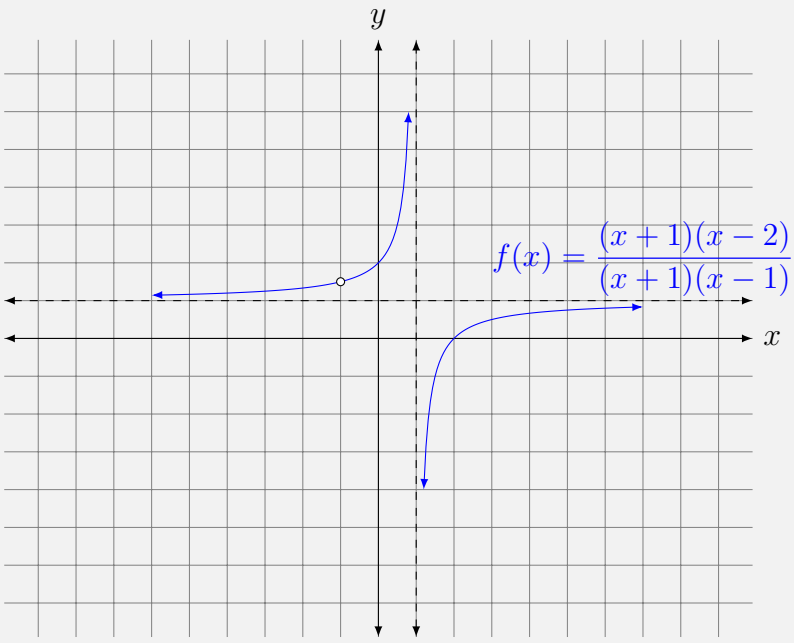
WRITE

Def.: The graph of a function $f : \mathbb{R} \to \mathbb{R}$ is the set of ordered pairs $(x, f(x))$. We say x is the independent variable and $y = f(x)$ is the dependent variable.

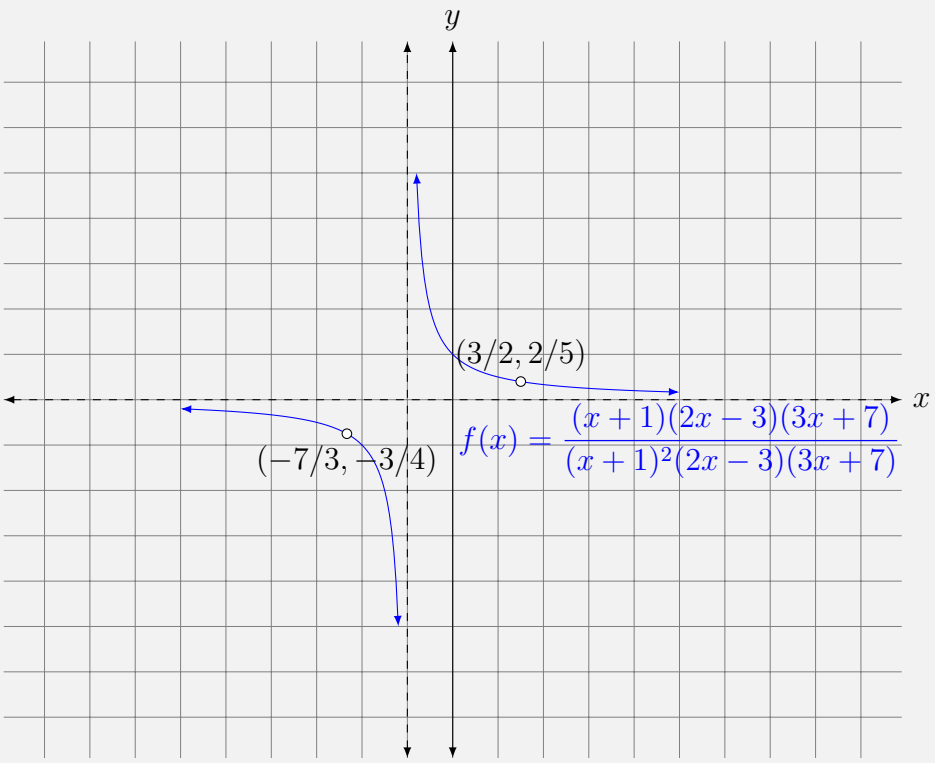
WRITE



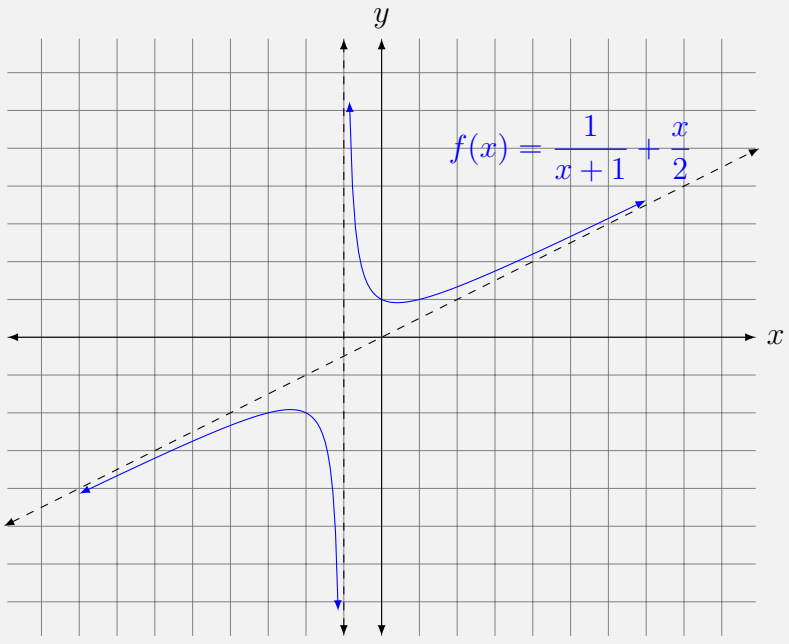
WRITE



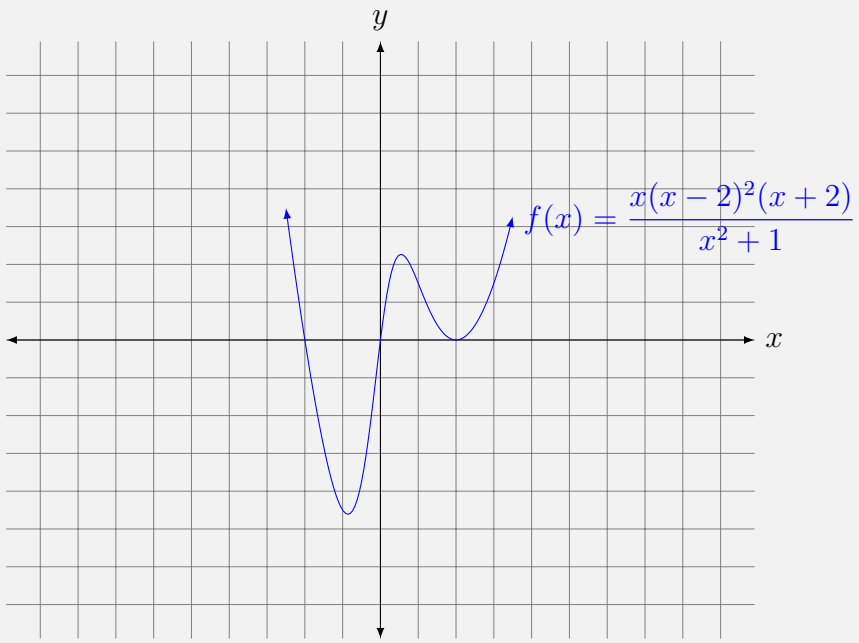
WRITE


y
x
(3/2, 2/5)
(−7/3, −3/4)
$f(x) = \dfrac{(x+1)(2x-3)(3x+7)}{(x+1)^2(2x-3)(3x+7)}$

WRITE


y
x
$f(x) = \dfrac{1}{x+1} + \dfrac{x}{2}$

WRITE



V. Even/Odd Functions

10:45am

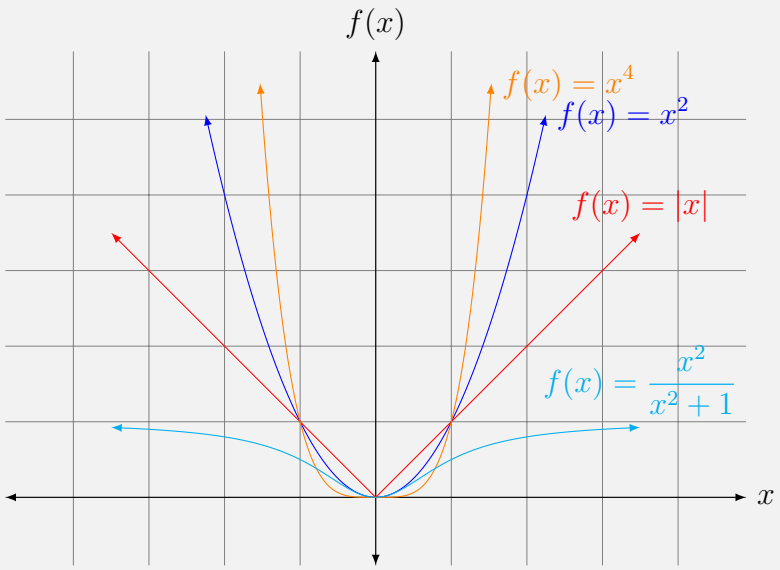
NOTE

- definition
- examples

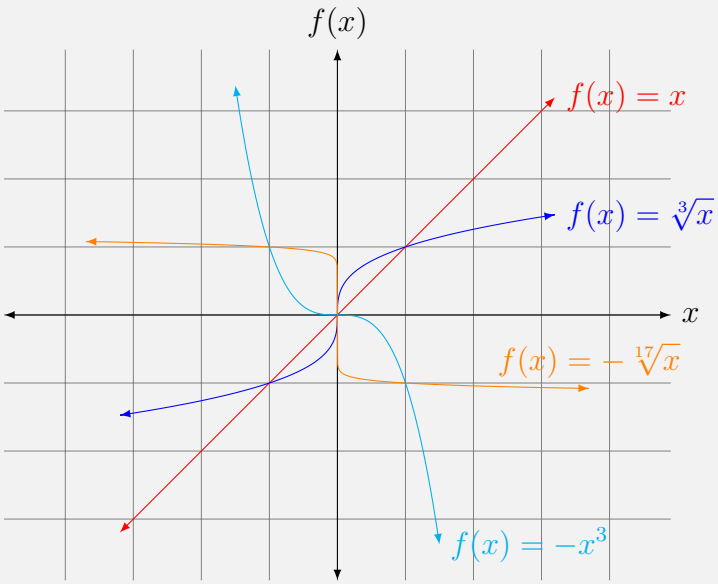
WRITE

Def.: A function $f : \mathbb{R} \to \mathbb{R}$ is even if $f(x) = f(-x)$ for all $x \in \mathbb{R}$.
Def.: A function $f : \mathbb{R} \to \mathbb{R}$ is odd if $f(x) = -f(-x)$ for all $x \in \mathbb{R}$.

WRITE


$f(x)$
$f(x) = x^4$
$f(x) = x^2$
$f(x) = |x|$
$f(x) = \dfrac{x^2}{x^2+1}$
x

WRITE


$f(x)$
$f(x) = x$
$f(x) = \sqrt[3]{x}$
x
$f(x) = -\sqrt[17]{x}$
$f(x) = -x^3$

VI. Increasing/Decreasing Functions and Sequences

10:55am

NOTE

- increasing/decreasing
- weakly increasing/weakly decreasing
- monotonic
- examples

WRITE

Def.: A sequence $\{a_n\}_{n=1}^{\infty}$ is increasing if $a_n > a_{n-1}$ for all $n > 1$. It is decreasing if $a_n < a_{n-1}$ for all $n > 1$.
Def.: A sequence $\{a_n\}_{n=1}^{\infty}$ is weakly increasing if $a_n \geq a_{n-1}$ for all $n > 1$. It is weakly decreasing if $a_n \leq a_{n-1}$ for all $n > 1$.
Def.: A sequence $\{a_n\}_{n=1}^{\infty}$ is monotonic if it is either weakly decreasing or weakly increasing.

NOTE

Functions can be increasing/decreasing/weakly increasing/weakly decreasing/monotonic on intervals of $\mathbb{R}$.

VII. Exercises

WRITE

(1) Please find the domain of the function
$$f(x) = \sqrt{3x+1}$$
(2) Is the function $f(x) = x^2 + 1$ even? Explain.
(3) Is the function $f(x) = \dfrac{1}{3x+1}$ odd? Explain.
(4) Please find the domain of the function
$$f(x) = \sqrt{\frac{x^2}{x^2 - 2x}}$$

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end
11:15am