MATH 1 LECTURE 5 WEDNESDAY 09-21-16

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Contents

I. Reminders/Announcements

start 10:10am Bartlett 105

Remarks

- Written HW#1 due TODAY
- Written HW#2 due next Wednesday
- WebWork HW04 due TODAY
- WebWork HW05 due Friday at 10:10am
- Quiz in BOX Kemeny 1st floor
- See m1f15 for old exams
- x-hour tomorrow
- MIDTERM1 is NEXT Thursday
- HAND OUT WORKSHEET FOR THE DAY

10:15am

II. Lagrange Interpolation

Examples

Lagrange Interpolation

Given $n+1$ data points $(x_1, y_1), (x_2, y_2), \ldots, (x_{n+1}, y_{n+1})$ we would like to construct a polynomial of degree n that "hits" all the data points.

- Suppose we have 2 data points (x_1, y_1) and (x_2, y_2). Let
$$f(x) = y_1 \frac{x - x_2}{x_1 - x_2} + y_2 \frac{x - x_1}{x_2 - x_1}.$$
Then $f(x_1) = y_1$ and $f(x_2) = y_2$ as desired.
- Suppose we have 3 data points $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3). Let
$$f(x) = y_1 \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)} + y_2 \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)} + y_3 \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}.$$
Then $f(x_i) = y_i$ for $i \in \{1, 2, 3\}$.

Exercises

- Suppose at the beginning of the month (day 1) you have 145 USD in your bank account and at the end of the month (day 30) you have 58 USD. Use Lagrange interpolation to estimate how much you have in the bank on day 23.

  Solution. The data points are $(1, 145)$ and $(30, 58)$. Lagrange interpolation yields the line
$$\begin{aligned} f(x) &= 145 \frac{x - 30}{1 - 30} + 58 \frac{x - 1}{30 - 1} \\ &= 145 \frac{x - 30}{-29} + 58 \frac{x - 1}{29} \\ &= -5(x - 30) + 2(x - 1) \\ &= -3x + 150 - 2 \\ &= -3x + 148. \end{aligned}$$
Now $f(23) = 79$.

- Now suppose we also know that on day 2 we still have 145 USD. Use Lagrange interpolation to estimate how much we have in the bank on day 23.

  Solution. The data points are $(1, 145), (2, 145), (30, 58)$. Lagrange interpolation yields the polynomial
$$\begin{aligned} f(x) &= 145 \frac{(x - 2)(x - 30)}{(1 - 30)(1 - 2)} + 145 \frac{(x - 1)(x - 30)}{(2 - 1)(2 - 30)} + 58 \frac{(x - 1)(x - 2)}{(30 - 1)(30 - 2)} \\ &= 5(x - 2)(x - 30) - \frac{145}{28}(x - 1)(x - 30) + \frac{1}{14}(x - 1)(x - 2). \end{aligned}$$
Now $f(23) = 95.5$.

10:25am

III. Compositions Revisited

Definition

A function is said to <u>decompose</u> if it can be written as the composition of other functions.

Examples

- $h(x) = (x-3)^2$ decomposes as $h = g \circ f$ where $g(x) = x^2$ and $f(x) = x - 3$.

MM: [Now do the first part of worksheet]

10:40am

IV. Transformations of Functions/Worksheet

IV.1. **Translations.**

Definition

Let $f : \mathbb{R} \to \mathbb{R}$ and $c > 0$. A <u>translation</u> by c is a transformation which horizontally or vertically shifts a function.

- $f(x-c)$ moves the function to the right by c
- $f(x+c)$ moves the function to the left by c
- $f(x)+c$ moves the function up by c
- $f(x)-c$ moves the function down by c

IV.2. **Reflections.**

Definition

Let $f : \mathbb{R} \to \mathbb{R}$. There are 2 types of reflections we consider.

- $-f(x)$ reflects $f(x)$ across the x-axis
- $f(-x)$ reflects $f(x)$ across the y-axis

IV.3. **Stretching/Contracting.**

Definition

Let $f : \mathbb{R} \to \mathbb{R}$ and $c > 1$. There are 4 ways to stretch/contract f that we consider.

- $cf(x)$ stretches $f(x)$ vertically by a factor of c
- $\frac{1}{c}f(x)$ contracts $f(x)$ vertically by a factor of c
- $f(cx)$ contracts $f(x)$ horizontally by a factor of c
- $f\left(\frac{1}{c}x\right)$ stretches $f(x)$ horizontally by a factor of c

IV.4. **Multiple Transformations.**

Remarks

Note that the above transformations are just special cases of compositions. But remember that composition is not commutative!

Examples

Let $f(x) = \sqrt{x}$. Suppose we want to reflect across the x-axis and shift up by 3. Does the order in which we do this matter?

- (First reflect then shift)

$$\sqrt{x} \mapsto -\sqrt{x} \mapsto -\sqrt{x} - 3$$

- (First shift then reflect)

$$\sqrt{x} \mapsto \sqrt{x} - 3 \mapsto -(\sqrt{x} - 3) = -\sqrt{x} + 3$$

MM: [finish worksheet]

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end
11:15am