Limit practice

Evaluate the following limits. Show all your work.

1.

$$\lim_{x\to 7} \frac{\ln(x-3)}{5^x}$$

Answer:

Since 7 in in the domain of $\ln(x-3)$ and 5^x, we may plug it in to get

$$\lim_{x\to 7} \frac{\ln(x-3)}{5^x} = \frac{\ln(4)}{5^7}$$

2.

$$\lim_{x\to 0} \sqrt{x^3+x^2}\sin\left(\frac{\pi}{x}\right)$$

Answer:

0 is not in the domain of $1/x$, and a trigonometric is involved. Observe that

$$-1 \le \sin\left(\frac{\pi}{x}\right) \le 1.$$

Since $\sqrt{x^3+x^2}$ is positive near 0 we may multiply all parts of the above inequality by it, to get

$$\sqrt{x^3+x^2} \le \sqrt{x^3+x^2}\sin\left(\frac{\pi}{x}\right) \le \sqrt{x^3+x^2}.$$

We observe that

$$\lim_{x\to 0} -\sqrt{x^3+x^2} = \lim_{x\to 0} \sqrt{x^3+x^2} = 0.$$

Thus by the squeeze theorem

$$\lim_{x\to 0} \sqrt{x^3+x^2}\sin\left(\frac{\pi}{x}\right) = 0$$

3.

$$\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h}$$

Answer:

Using algebra to simplify, we get

$$\frac{(3+h)^{-1}-3^{-1}}{h} = \frac{1}{h}\left(\frac{1}{3+h}-\frac{1}{3}\right) = \frac{1}{h}\left(\frac{3}{3(3+h)}-\frac{3+h}{3(3+h)}\right)$$
$$\frac{1}{h}\left(\frac{3-(3+h)}{3(3+h)}\right) = \frac{1}{h}\left(\frac{-h}{3(3+h)}\right) = \frac{-1}{3(3+h)}$$

Which is a rational function where 0 is in the domain. Thus by the rational limit theorem.

$$\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h} = \frac{-1}{9}$$

4.

$$\lim_{t\to\frac{\pi}{2}}\left(\left(t-\frac{\pi}{2}\right)\sin(t)\right)$$

Answer:

$\pi/2$ is in the domain of the product of continuous functions, so the limit is

$$\lim_{t\to\frac{\pi}{2}}\left(\left(t-\frac{\pi}{2}\right)\sin(t)\right)=\left(\left(\frac{\pi}{2}-\frac{\pi}{2}\right)\sin(\frac{\pi}{2})\right)=0$$

5.

$$\lim_{x\to 0^-}\frac{|x|}{x}$$

Answer:

Since we are approaching from the left, we only need to consider negative values. Observe that when $x<0$, that $|x|=-x$. Thus

$$\lim_{x\to 0^-}\frac{|x|}{x}=\lim_{x\to 0^-}\frac{-x}{x}=\lim_{x\to 0^-}-1=-1$$

6.

$$\lim_{x\to 0^+}\left(\sqrt{x}[1+\sin^2(2\pi/x)]\right)$$

Answer:

0 is not in the domain of $2\pi/x$, and a trigonometric is involved. Observe that

$$1\leq 1+\sin^2(2\pi/x)\leq 2.$$

Since $\sqrt{x}$ is positive near 0 on the right hand side, we may multiply all parts of the above inequality by it, to get

$$\sqrt{x}\leq\sqrt{x}[1+\sin^2(2\pi/x)]\leq 2\sqrt{x}.$$

We observe that

$$\lim_{x\to 0^+}\sqrt{x}=\lim_{x\to 0^+}2\sqrt{x}=0.$$

Thus by the squeeze theorem

$$\lim_{x\to 0^+}\left(\sqrt{x}[1+\sin^2(2\pi/x)]\right)=0$$