Math 1 Lecture 14

Dartmouth College

Wednesday 10-12-16

Contents

Reminders/Announcements

- WebWork due Friday
- x-hour tomorrow
- Exam#2 is next Thursday 10/20/16 and will cover material from Trigonometry up to and NOT including derivatives

More Examples

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We can manipulate the functions in an algebraic way to make limit computations more apparent.

Continuity at a point

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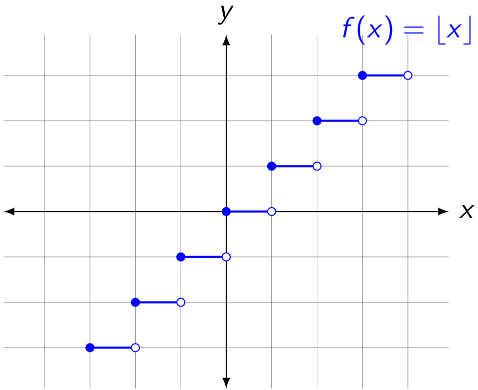
Similarly, a function $f(x)$ is **left continuous** at a number a if

$$\lim_{x \to a^-} f(x) = f(a)$$

and is **right continuous** at a number a if

$$\lim_{x \to a^+} f(x) = f(a).$$

Consider the floor function $f(x) = \lfloor x \rfloor$.



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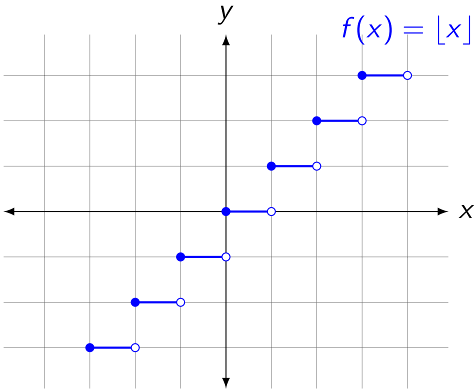

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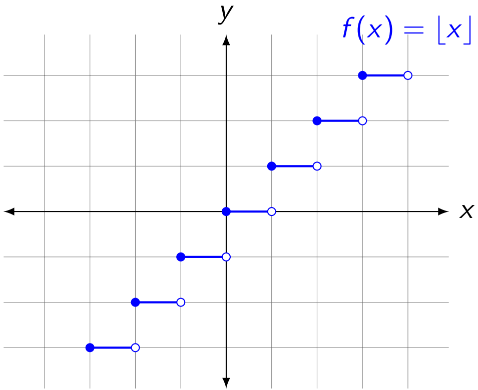

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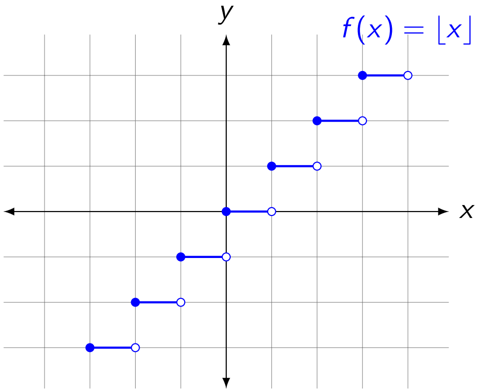

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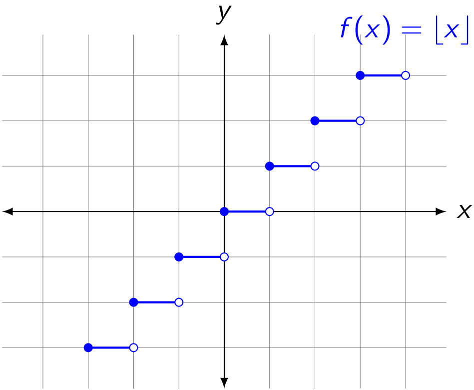

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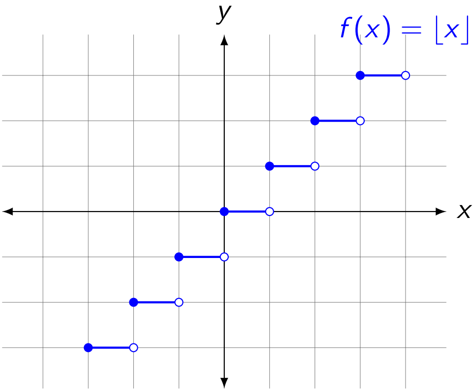

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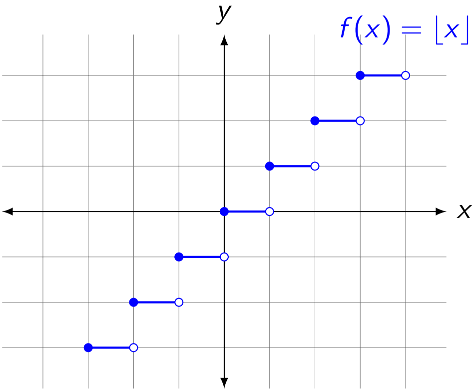

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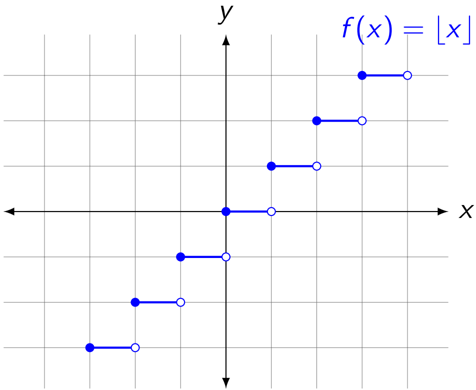

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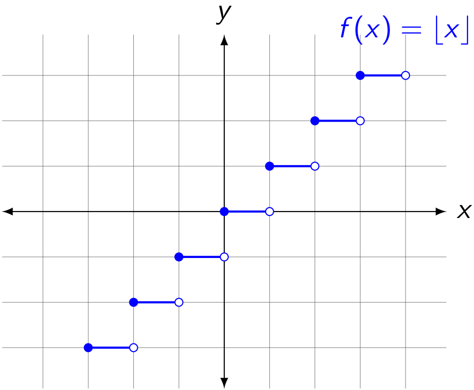

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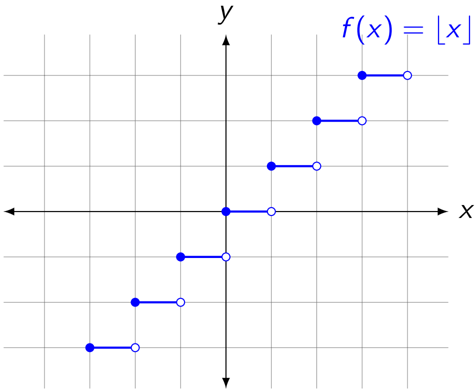

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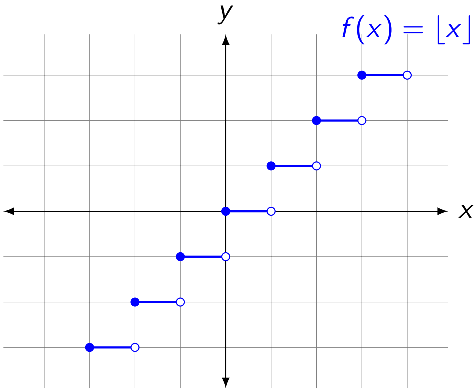

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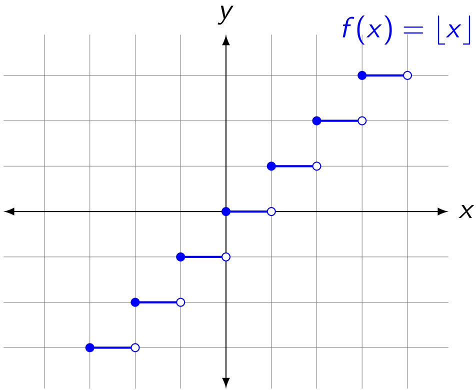

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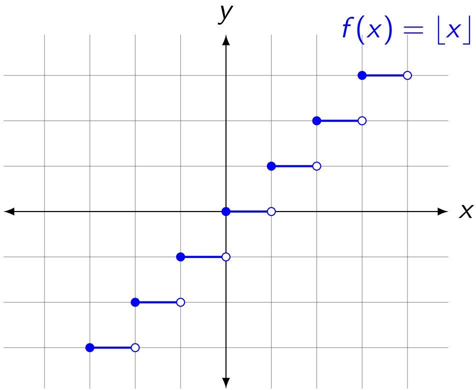

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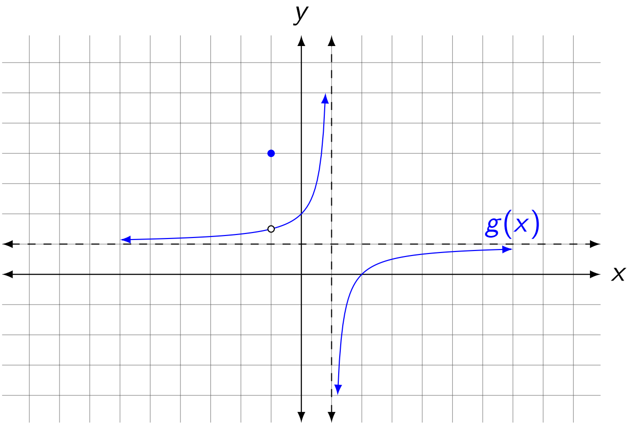
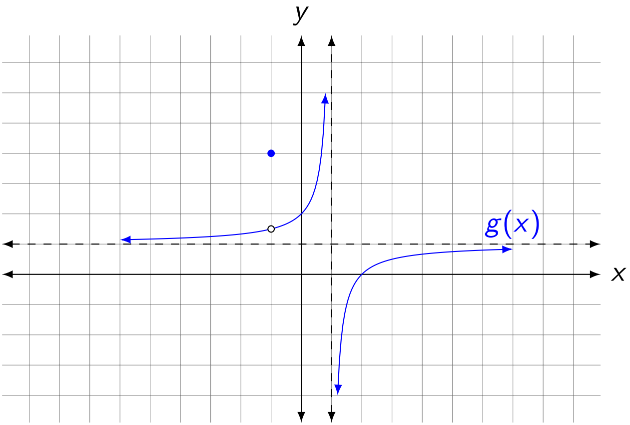
Since this is a bit technical, we now give some examples.

Removable discontinuity



Where does g have a removable discontinuity?

Removable discontinuity



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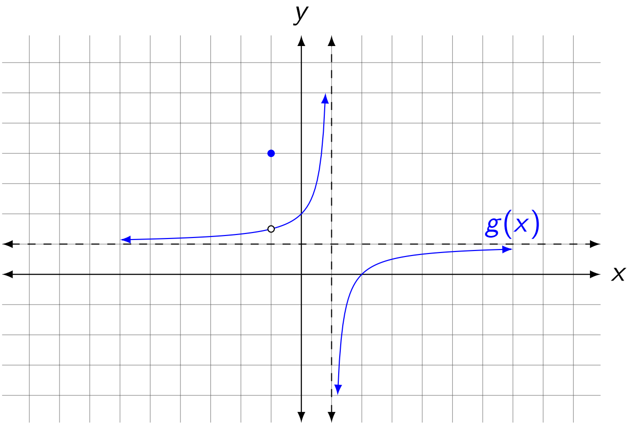
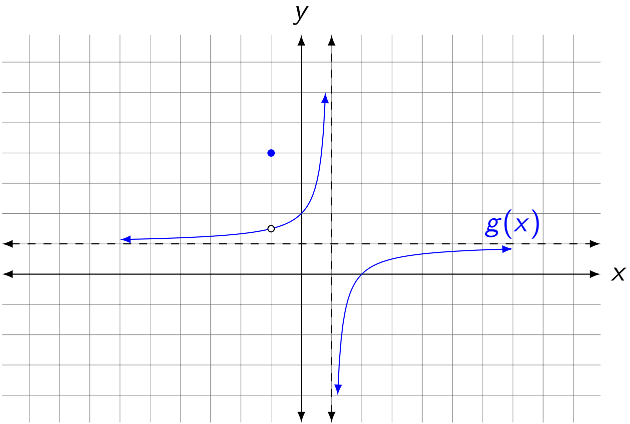


Where does g have a removable discontinuity? g has a removable discontinuity at -1. We call this type of discontinuity **removable** since it could be made continuous by "adding a single point".

Infinite discontinuity



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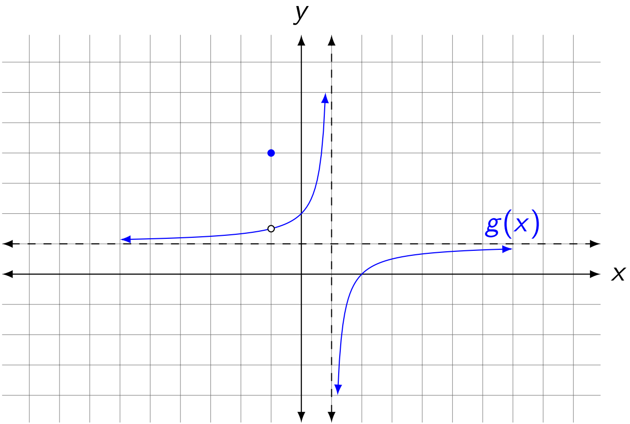
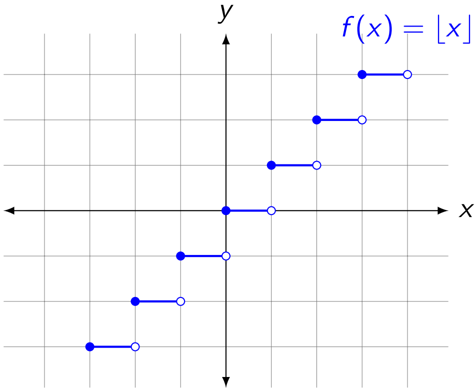


Where does g have an infinite discontinuity? g has an infinite discontinuity at 1.

Jump discontinuity



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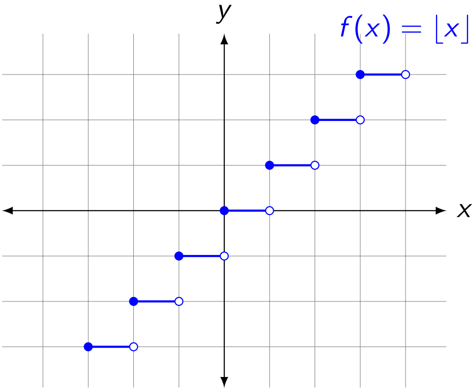
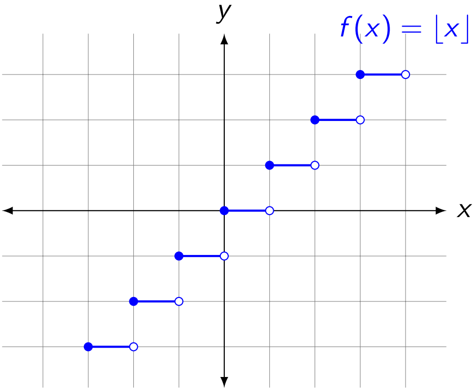


Where is f discontinuous?

Jump discontinuity



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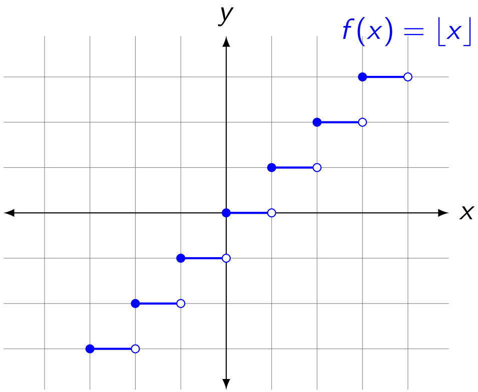

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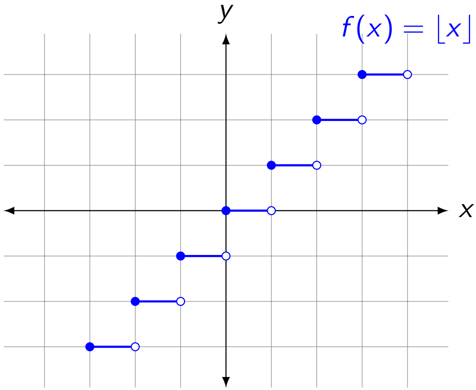

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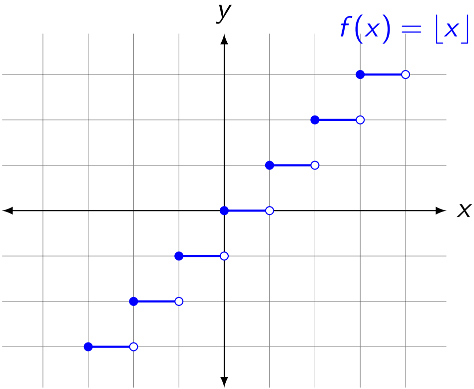

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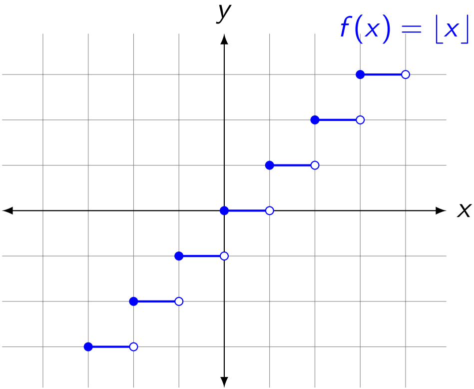

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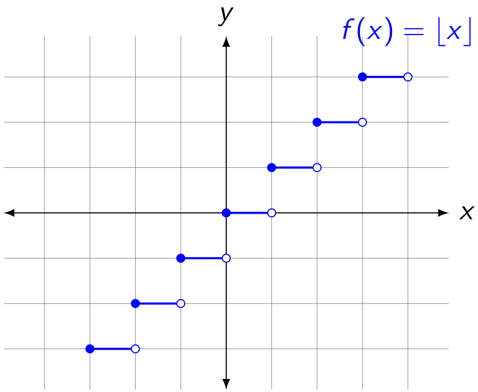

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$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right).$$

Compute $\lim_{x \to 2} \sqrt{\frac{2x^2+1}{3x-2}}$.

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$$\begin{aligned}
\lim_{x\to 2}\sqrt{\frac{2x^2+1}{3x-2}} &= \sqrt{\lim_{x\to 2}\frac{2x^2+1}{3x-2}} \\
&= \sqrt{\frac{2\cdot 2^2+1}{3\cdot 2-2}} \\
&= \sqrt{\frac{9}{4}} \\
&= 3/2.
\end{aligned}$$

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Polynomials are continuous on $(-\infty, \infty)$.
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The functions $\sqrt[n]{}$ are continuous on their domains.
Trigonometric functions are continuous on their domains.

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Solution:
g is a composition of functions that are all continuous on their domains. Thus g is also continuous on its domain.

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g is a composition of functions that are all continuous on their domains. Thus g is also continuous on its domain. What is the domain of g?

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Intermediate Value Theorem

Suppose f is continuous on the interval $[a, b]$ and $f(a) \neq f(b)$. Let $N \in [f(a), f(b)]$. Then there exists $c \in (a, b)$ such that $f(c) = N$.

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Proof.

Draw a picture! $\square$

Use the intermediate value theorem to show that there is a root of the equation

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Solution:
Let $f(x) = 4x^3 - 6x^2 + 3x - 2$ and apply IVT with $[a, b] = [1, 2]$ and $N = 0$.

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Let

$$f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4 \end{cases}.$$

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Solution: f is continuous everywhere.

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$$f(x) = \frac{5(x-5)^3}{(x+3)(x-2)x}.$$

Why $x - 5$? Well, it doesn't really matter except that we don't want the numerator to be zero when the denominator is. So 5 could have been anything except $-3, 0, 2$.

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