Math 1 Lecture 18

Dartmouth College

Friday 10-21-16

Contents

Reminders/Announcements

- WebWork due Monday
- Quiz Monday

Last Time

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Today we will start to see how we can use rules about derivatives to compute them more efficiently.

Derivatives of Constant Functions

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It's zero! We summarize this as follows:

$$\boxed{\frac{d}{dx}(c) = 0}$$

Derivatives of Power Functions

Suppose we have a power function $f(x) = x^r$ where r is a fixed real number.

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$$\boxed{\frac{d}{dx}(x^r) = r \cdot x^{r-1}}.$$

Derivatives of Sums and Constant Multiples

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$$\boxed{\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))}$$

$$\boxed{\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}(f(x))}.$$

To Summarize...

- $$\frac{d}{dx}(c) = 0$$
- $$\frac{d}{dx}(x^r) = rx^{r-1}$$
- $$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$
- $$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$$

Computation Examples

Find the derivative of each function given below.

1. $f(t) = 2t^3 - 3t^2 - 4t$
2. $y = x^{5/3} - x^{2/3}$
3. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$
4. $f(x) = \pi^4$
5. $u = \left(\dfrac{1}{t} - \dfrac{1}{\sqrt{t}}\right)^2$

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Solution:
$v(t) = s'(t) = 3t^2 - 3$, so the velocity of the particle after 2 seconds is $v(2) = s'(2) = 3 \cdot 2^2 - 3 = 9$ meters per second.

Biologists have proposed a cubic polynomial to model the length L of Alaskan rockrish at age A:

$$L = 0.0155A^3 - 0.372A^2 + 3.95A + 1.21$$

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This is just notation asking you to first compute the function L' and then plug in 12 to get $L'(12)$. Using derivative rules we see that

$$L'(A) = 3(0.0155)A^2 - 2(0.372)A + 3.95 = 0.0465A^2 - 0.744A + 3.95.$$

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Now $L'(12) = 1.718$. This number represents the rate at which the length of the fish is changing when it is 12 years old.

Derivatives of Exponential Functions

Let $f(x) = a^x$ for some $a > 0$.

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Let $f(x) = a^x$ for some $a > 0$. Then by definition we have

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} \\
&= \lim_{h \to 0} \frac{a^x \cdot a^h - a^x}{h} \\
&= \lim_{h \to 0} \left(a^x \cdot \frac{a^h - 1}{h} \right) \\
&= \lim_{h \to 0} (a^x) \cdot \lim_{h \to 0} \left(\frac{a^h - 1}{h} \right) \\
&= a^x \lim_{h \to 0} \frac{a^h - 1}{h} \\
&= a^x \cdot f'(0).
\end{aligned}$$

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$$1 = f'(0) = \lim_{h \to 0} \frac{e^{0+h} - e^0}{h} = \lim_{h \to 0} \frac{e^h - 1}{h}.$$

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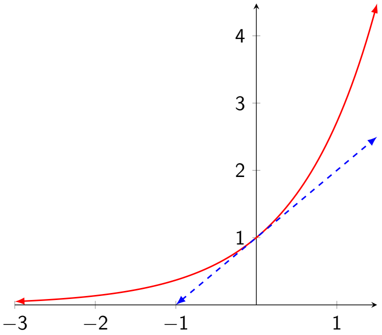

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by definition of the number e. Thus $f'(x) = f(x)$. We've constructed a function whose derivative is itself! In Summary,

$$\boxed{\frac{d}{dx}(e^x) = e^x}.$$

Compute the derivative of $y = 3e^{x+2} + x^e$.

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Solution:

$$
\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{dx}\left(3e^{x+2} + x^e\right) \\
&= \frac{dy}{dx}(3 \cdot e^2 \cdot e^x + x^e) \\
&= \frac{dy}{dx}(3 \cdot e^2 \cdot e^x) + \frac{dy}{dx}(x^e) \\
&= 3e^2 \cdot \frac{dy}{dx}(e^x) + \frac{dy}{dx}(x^e) \\
&= 3e^2 e^x + ex^{e-1}.
\end{aligned}
$$

Consider the function

$$f(x) = \frac{5}{3}x^3 - \frac{5}{2}x^2 - 10x.$$

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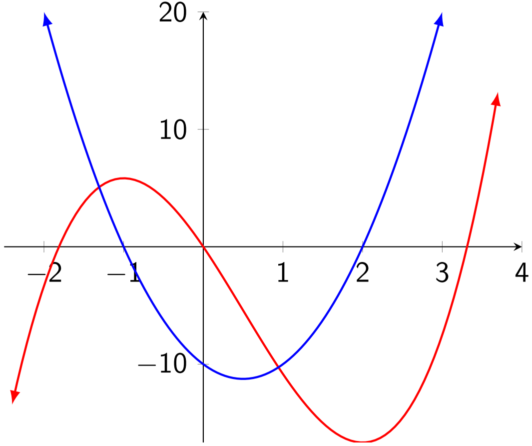
- Find $f'(x)$.
- Solve $f'(x) = 0$.
- Find the interval(s) where $f'(x) > 0$.
- Find the interval(s) where $f'(x) < 0$.


20
10
−10
−2
−1
1
2
3
4

Have a great weekend!