Math 1 Lecture 19

Dartmouth College

Monday 10-24-16

Contents

Reminders/Announcements

- WebWork due Wednesday
- Written Homework due Wednesday
- x-hour problem session drop in Thursday

- $$\frac{d}{dx}(c) = 0$$

- $$\frac{d}{dx}(x^r) = rx^{r-1}$$

- $$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

- $$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$$

- $$\frac{d}{dx}(e^x) = e^x, \quad \text{where } e = 2.7182818284590\ldots$$

- $$\frac{d}{dx}(c) = 0$$

- $$\frac{d}{dx}(x^r) = rx^{r-1}$$

- $$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

- $$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$$

- $$\frac{d}{dx}(e^x) = e^x, \quad \text{where } e = 2.7182818284590\ldots$$

But there are more properites!

- $$\frac{d}{dx}(c) = 0$$

- $$\frac{d}{dx}(x^r) = rx^{r-1}$$

- $$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

- $$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$$

- $$\frac{d}{dx}(e^x) = e^x, \quad \text{where } e = 2.7182818284590\ldots$$

But there are more properites! Of course. . . 😋

The Product Rule for Derivatives

Suppose we want to compute the derivative of a product of functions $h(x) = f(x)g(x) \ldots$

The Product Rule for Derivatives

Suppose we want to compute the derivative of a product of functions $h(x) = f(x)g(x)$... Unfortunately we cannot just take the product of the derivatives...

The Product Rule for Derivatives

Suppose we want to compute the derivative of a product of functions $h(x) = f(x)g(x)$... Unfortunately we cannot just take the product of the derivatives... ☹ But all is not lost, we can compute $h'(x)$ using the product rule:

The Product Rule for Derivatives

Suppose we want to compute the derivative of a product of functions $h(x) = f(x)g(x)$... Unfortunately we cannot just take the product of the derivatives... But all is not lost, we can compute $h'(x)$ using the product rule:

$$\boxed{h'(x) = f(x)g'(x) + f'(x)g(x)}$$

Let $h(x) = f(x)g(x)$ and 😇 $= f(x+h)g(x)$.

Let $h(x) = f(x)g(x)$ and $\text{😇} = f(x+h)g(x)$. Then using the definition we have...

$$
\begin{aligned}
h'(x) &= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x+h) - \text{😇} + \text{😇} - f(x)g(x)}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x+h) - \text{😇}}{h} + \lim_{h\to 0} \frac{\text{😇} - f(x)g(x)}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)\Big(g(x+h) - g(x)\Big)}{h} + \lim_{h\to 0} \frac{g(x)\Big(f(x+h) - f(x)\Big)}{h} \\
&= \left(\lim_{h\to 0} f(x+h)\right) g'(x) + \left(\lim_{h\to 0} g(x)\right) f'(x) \\
&= f(x)g'(x) + g(x)f'(x) \\
&= f(x)g'(x) + f'(x)g(x).
\end{aligned}
$$

The Quotient Rule for Derivatives

Similarly, we should not expect the derivative of a quotient $h(x) = f(x)/g(x)$ to be the quotient of the derivatives...

The Quotient Rule for Derivatives

Similarly, we should not expect the derivative of a quotient $h(x) = f(x)/g(x)$ to be the quotient of the derivatives... But we can find the derivative of a quotient as follows:

The Quotient Rule for Derivatives

Similarly, we should not expect the derivative of a quotient $h(x) = f(x)/g(x)$ to be the quotient of the derivatives... But we can find the derivative of a quotient as follows:

$$\boxed{h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}}$$

The Quotient Rule for Derivatives

Similarly, we should not expect the derivative of a quotient $h(x) = f(x)/g(x)$ to be the quotient of the derivatives... But we can find the derivative of a quotient as follows:

$$\boxed{h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{\Big(g(x)\Big)^2}}$$

Some people say "low-dee-high-minus-high-dee-low" and then something about the thing below...

The Quotient Rule for Derivatives

Similarly, we should not expect the derivative of a quotient $h(x) = f(x)/g(x)$ to be the quotient of the derivatives... But we can find the derivative of a quotient as follows:

$$\boxed{h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{\Big(g(x)\Big)^2}}$$

Some people say "low-dee-high-minus-high-dee-low" and then something about the thing below... maybe that's helpful to remember the quotient rule.

Let $h(x) = f(x)/g(x)$ and 😇 $= f(x)g(x)$.

Let $h(x) = f(x)/g(x)$ and $\text{😇} = f(x)g(x)$. Then

$$
\begin{aligned}
h'(x) &= \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} \\
&= \lim_{h \to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{hg(x+h)g(x)} \\
&= \lim_{h \to 0} \frac{f(x+h)g(x) - \text{😇} + \text{😇} - f(x)g(x+h)}{hg(x+h)g(x)} \\
&\vdots \\
&= \frac{g(x)f'(x) - f(x)g'(x)}{\Big(g(x)\Big)^2}.
\end{aligned}
$$

Let $h(x) = f(x)/g(x)$ and $\text{😇} = f(x)g(x)$. Then

$$
\begin{aligned}
h'(x) &= \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{hg(x+h)g(x)} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x) - \text{😇} + \text{😇} - f(x)g(x+h)}{hg(x+h)g(x)} \\
&\;\vdots \\
&= \frac{g(x)f'(x) - f(x)g'(x)}{\big(g(x)\big)^2}.
\end{aligned}
$$

Maybe you can fill in the details...

Computational Examples

1. $y = \dfrac{1}{t^3 + 2t^2 - 1}$
2. $y = \dfrac{\sqrt{x}}{2 + x}$
3. $A(v) = v^{2/3}(2v^2 + 1 - v^{-2})$
4. $f(x) = \dfrac{x}{x + \frac{9}{x}}$
5. $f(x) = \dfrac{ax + b}{cx + d}$, for $a, b, c, d \in \mathbb{R}$

Computational Examples

1. $y = \dfrac{1}{t^3 + 2t^2 - 1}$
2. $y = \dfrac{\sqrt{x}}{2 + x}$
3. $A(v) = v^{2/3}(2v^2 + 1 - v^{-2})$
4. $f(x) = \dfrac{x}{x + \frac{9}{x}}$
5. $f(x) = \dfrac{ax + b}{cx + d}$, for $a, b, c, d \in \mathbb{R}$

Find the derivatives of course...

Higher Derivatives

Given a function $f(x)$ we have discussed how we go about defining a new function $f'(x)$.

Higher Derivatives

Given a function $f(x)$ we have discussed how we go about defining a new function $f'(x)$. Where is f' defined?

Higher Derivatives

Given a function $f(x)$ we have discussed how we go about defining a new function $f'(x)$. Where is f' defined? What's an example of a function that isn't differentiable everywhere?

Higher Derivatives

Given a function $f(x)$ we have discussed how we go about defining a new function $f'(x)$. Where is f' defined? What's an example of a function that isn't differentiable everywhere? Nothing is stopping us from repeating this process.

Higher Derivatives

Given a function $f(x)$ we have discussed how we go about defining a new function $f'(x)$. Where is f' defined? What's an example of a function that isn't differentiable everywhere? Nothing is stopping us from repeating this process. That is, taking the derivative of the derivative...

Higher Derivatives

Given a function $f(x)$ we have discussed how we go about defining a new function $f'(x)$. Where is f' defined? What's an example of a function that isn't differentiable everywhere? Nothing is stopping us from repeating this process. That is, taking the derivative of the derivative... OK, suppose we want to find the "second derivative" of f.

Higher Derivatives

Given a function $f(x)$ we have discussed how we go about defining a new function $f'(x)$. Where is f' defined? What's an example of a function that isn't differentiable everywhere? Nothing is stopping us from repeating this process. That is, taking the derivative of the derivative... OK, suppose we want to find the "second derivative" of f. How should we define such a function?

Higher Derivatives

Given a function $f(x)$ we have discussed how we go about defining a new function $f'(x)$. Where is f' defined? What's an example of a function that isn't differentiable everywhere? Nothing is stopping us from repeating this process. That is, taking the derivative of the derivative... OK, suppose we want to find the "second derivative" of f. How should we define such a function? Well...

Higher Derivatives

Given a function $f(x)$ we have discussed how we go about defining a new function $f'(x)$. Where is f' defined? What's an example of a function that isn't differentiable everywhere? Nothing is stopping us from repeating this process. That is, taking the derivative of the derivative... OK, suppose we want to find the "second derivative" of f. How should we define such a function? Well...

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

whenever this limit exists.

Higher Derivatives

Given a function $f(x)$ we have discussed how we go about defining a new function $f'(x)$. Where is f' defined? What's an example of a function that isn't differentiable everywhere? Nothing is stopping us from repeating this process. That is, taking the derivative of the derivative. . . OK, suppose we want to find the "second derivative" of f. How should we define such a function? Well. . .

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

whenever this limit exists. Similarly, we can define higher derivatives

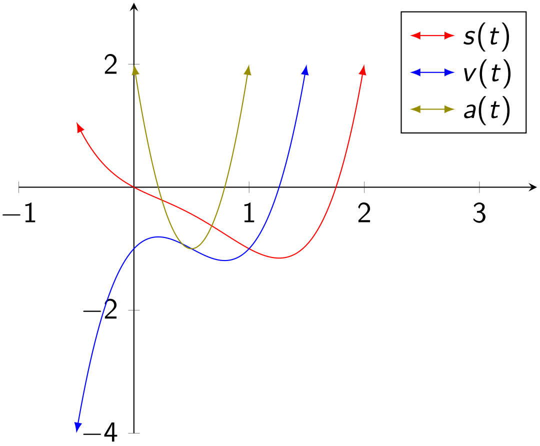
$$\boxed{\frac{d^n}{dx^n}(f(x)) = f^{(n)}(x) = \lim_{h \to 0} \frac{f^{(n-1)}(x+h) - f^{(n-1)}(x)}{h}}$$

which are valid on the appropriate domains.

The equation of motion of a particle is $s(t) = t^4 - 2t^3 + t^2 - t$, where s is in meters and t is in seconds.

The equation of motion of a particle is $s(t) = t^4 - 2t^3 + t^2 - t$, where s is in meters and t is in seconds.

- Find the velocity and acceleration as functions of t.
- Find the velocity after 1 second.
- Find the acceleration after 1 second.



2
−2
−4
−1
1
2
3
s(t)
v(t)
a(t)

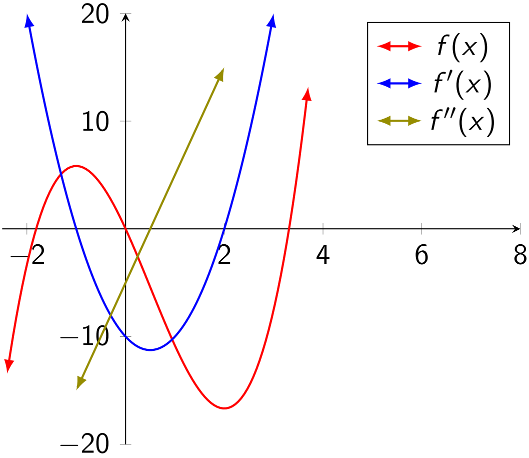
Consider the function

$$f(x) = \frac{5}{3}x^3 - \frac{5}{2}x^2 - 10x.$$

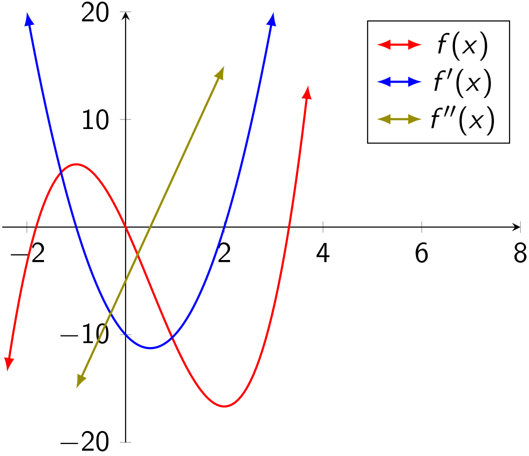
Consider the function

$$f(x) = \frac{5}{3}x^3 - \frac{5}{2}x^2 - 10x.$$

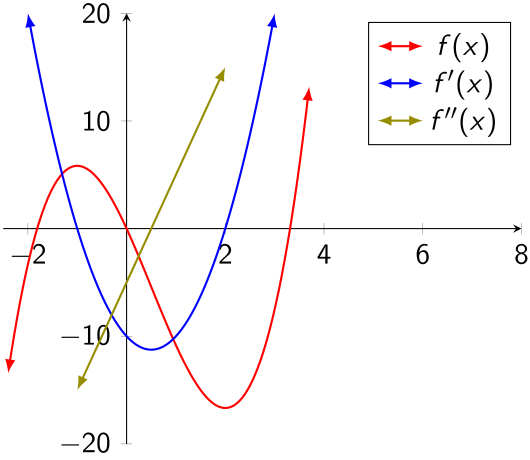
- Find $f'(x)$.
- Solve $f'(x) = 0$.
- Find the interval(s) where $f'(x) > 0$.
- Find the interval(s) where $f'(x) < 0$.
- Find $f''(x)$.
- Solve $f''(x) = 0$.
- Find the interval(s) where $f''(x) > 0$.
- Find the interval(s) where $f''(x) < 0$.



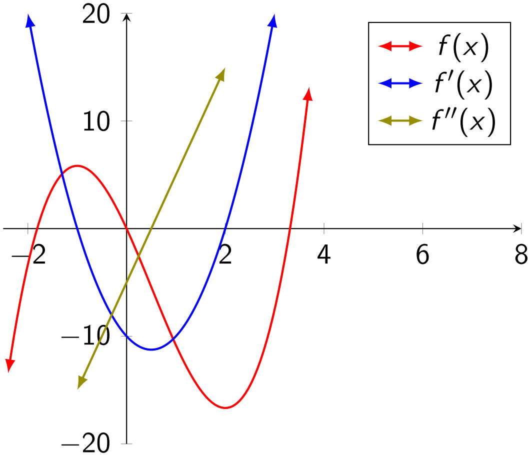
20
10
−10
−20
−2
2
4
6
8
f(x)
f′(x)
f″(x)



What is $f'''(x)$?



What is $f'''(x)$? What is $f''''(x)$?



What is $f'''(x)$? What is $f''''(x)$? What is $f^{(n)}(x)$ for $n \geq 4$?

Suppose that $f(4) = 2$, $g(4) = 5$, $f'(4) = 6$, and $g'(4) = -3$.

Suppose that $f(4) = 2$, $g(4) = 5$, $f'(4) = 6$, and $g'(4) = -3$. Find $h'(4)$ for the following functions $h(x)$.

Suppose that $f(4) = 2$, $g(4) = 5$, $f'(4) = 6$, and $g'(4) = -3$. Find $h'(4)$ for the following functions $h(x)$.

1. $h(x) = 3f(x) + 8g(x)$
2. $h(x) = f(x)g(x)$
3. $h(x) = \dfrac{f(x)}{g(x)}$
4. $h(x) = \dfrac{g(x)}{f(x) + g(x)}$

For what values of x does the graph of $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent line?

For what values of x does the graph of $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent line?

Compute

$$\frac{d^2}{dx^2}(f(x)), \quad \frac{d^3}{dx^3}(f(x)), \text{ and } \quad \frac{d^4}{dx^4}(f(x)).$$

Suppose $h(2) = 4$ and $h'(2) = -3$. Compute the following values:

1.

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\bigg|_{x=2}$$

2.

$$\frac{d^2}{dx^2}\left(\frac{h(x)}{x}\right)\bigg|_{x=2}$$