Math 1 Lecture 21

Dartmouth College

Friday 10-28-16

Contents

Reminders/Announcements

- WebWork due Monday
- Quiz Monday
- Written HW due Wednesday

Last Time

- Derivatives of Trigonometric Functions

The Chain Rule

We have yet to talk about how derivatives behave with regards to compositions of functions.

The Chain Rule

We have yet to talk about how derivatives behave with regards to compositions of functions. Let $h(x) = (f \circ g)(x) = f(g(x))$ and suppose g is differentiable at x and f is differentiable at $g(x)$.

The Chain Rule

We have yet to talk about how derivatives behave with regards to compositions of functions. Let $h(x) = (f \circ g)(x) = f(g(x))$ and suppose g is differentiable at x and f is differentiable at $g(x)$. Then the **chain rule** states that

$$\boxed{h'(x) = f'(g(x)) \cdot g'(x)}.$$

A proof of this is beyond the scope of this course, but learning how to apply this will enable us to compute many more examples.

Alternative Formulation

We can also state the chain rule using slightly different notation

Alternative Formulation

We can also state the chain rule using slightly different notation which may or may not be helpful!

Alternative Formulation

We can also state the chain rule using slightly different notation which may or may not be helpful!

Let $y = f(u)$ and $u = g(x)$ (both differentiable). Then an alternative formulation of the chain rule says that

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}.$$

Compute the derivative of $h(x) = e^{x^3 - x}$.

Compute the derivative of $h(x) = e^{x^3 - x}$.

Solution:

We can write $h(x) = f(g(x))$ with $f(x) = e^x$ and $g(x) = x^3 - x$.

Compute the derivative of $h(x) = e^{x^3 - x}$.

Solution:

We can write $h(x) = f(g(x))$ with $f(x) = e^x$ and $g(x) = x^3 - x$. By basic derivative rules we have

$$f'(x) = e^x$$
$$g'(x) = 3x^2 - 1.$$

The chain rule allows us to put this all together to find $h'(x)$.

Compute the derivative of $h(x) = e^{x^3 - x}$.

Solution:

We can write $h(x) = f(g(x))$ with $f(x) = e^x$ and $g(x) = x^3 - x$. By basic derivative rules we have

$$\begin{aligned} f'(x) &= e^x \\ g'(x) &= 3x^2 - 1. \end{aligned}$$

The chain rule allows us to put this all together to find $h'(x)$. By the chain rule,

$$\begin{aligned} h'(x) &= f'(g(x)) \cdot g'(x) \\ &= e^{g(x)} \cdot (3x^2 - 1) \\ &= e^{x^3 - x} \cdot (3x^2 - 1). \end{aligned}$$

Compute the derivative of $h(x) = \sqrt{3x - 5}$.

Compute the derivative of $h(x) = \sqrt{3x - 5}$.

Solution:

We can write $h(x) = f(g(x))$ with $f(x) = \sqrt{x}$ and $g(x) = 3x - 5$.

Compute the derivative of $h(x) = \sqrt{3x-5}$.

Solution:

We can write $h(x) = f(g(x))$ with $f(x) = \sqrt{x}$ and $g(x) = 3x - 5$. By basic derivative rules we have

$$f'(x) = \frac{1}{2\sqrt{x}}$$
$$g'(x) = 3.$$

The chain rule allows us to put this all together to find $h'(x)$.

Compute the derivative of $h(x) = \sqrt{3x-5}$.

Solution:

We can write $h(x) = f(g(x))$ with $f(x) = \sqrt{x}$ and $g(x) = 3x - 5$. By basic derivative rules we have

$$\begin{aligned} f'(x) &= \frac{1}{2\sqrt{x}} \\ g'(x) &= 3. \end{aligned}$$

The chain rule allows us to put this all together to find $h'(x)$. By the chain rule,

$$\begin{aligned} h'(x) &= f'(g(x)) \cdot g'(x) \\ &= \frac{1}{2\sqrt{g(x)}} \cdot 3 \\ &= \frac{1}{2\sqrt{3x-5}} \cdot 3 \\ &= \frac{3}{2\sqrt{3x-5}}. \end{aligned}$$

Exercises (Basic)

1. $y = \sin(\pi x)$
2. $y = \sin(\cos(x))$
3. $y = \left(x + \frac{1}{x^2}\right)^{\sqrt{7}}$
4. $y = (1 - x^{-1})^{-1}$

Exercises (Examish)

1. $f(x) = \sin(e^x)$
2. $g(x) = \tan(x^2 - 5)$
3. $h(x) = e^{\sin(2x)}$
4. $y = \sqrt[5]{x^2 - \sin(x)}$
5. $y = \frac{1}{\sqrt[5]{x^2 - \sin(x)}}$
6. $s(t) = \sec(t^2 - 3)$
7. $x(t) = 2t + \frac{1}{\sqrt{3t^2 + 5t + 7}}$
8. $y(t) = \tan(1 + \sin(t^2))$

Exercises (Hard... or maybe just plain annoying?)

1. $h(x) = (x^3 - x)e^{x^2}\cos(2x - 5)$
2. $k(x) = \frac{e^{x^2}}{(x^3 - x)\cos(2x - 5)}$
3. $y = \left(\sin\left(\cos\left(\sqrt{\sin(\pi x)}\right)\right)\right)^2$

Practice Quiz!

Find the derivatives of the functions given below.

1. $f(x) = e^{2x+1}\cos(x)$
2. $g(x) = \cos(e^x)$
3. $h(x) = \sin(x^3 - x)$