PRACTICE PROBLEMS

(1) Compute the following expressions:

(a) $\sin(\frac{7\pi}{4})$

Solution The angle is in the fourth quadrant: $\frac{3\pi}{2} = \frac{6\pi}{4} < \frac{7\pi}{4} < \frac{8\pi}{4} = 2\pi$, so the y-coordinate of the point corresponding to this angle is negative. Thus $\sin(\frac{7\pi}{4}) = -\frac{\sqrt{2}}{2}$, since the leftover angle is $2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$, and $\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$.

(b) $\arctan(-\sqrt{3})$

Solution We are looking for an angle θ in the range $(-\pi/2, \pi/2)$ so that $\tan(\theta) = -\sqrt{3}$. Since the value of arctangent is negative, the angle has to be in the range $(-\pi/2, 0)$. This angle is $-\frac{\pi}{3}$.

(c) $\arccos(\cos(\frac{7\pi}{4}))$

Solution We cannot use the cancellation laws, since $\frac{7\pi}{4}$ is not in the range $[0, \pi]$ of $\arccos(x)$.

Instead, $\arccos(\cos(\frac{7\pi}{4})) = \arccos(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$.

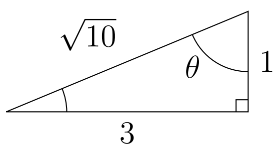
(d) $\arctan(\tan(\frac{\pi}{6}))$

Solution In this case we can use the cancellation laws, since $\frac{\pi}{6}$ is in the range $(-\pi/2, \pi/2)$ of $\arctan(x)$, and $\arctan(\tan(\frac{\pi}{6})) = \frac{\pi}{6}$.

(2) Compute the following expressions:

(a) $\sec(\arctan(3))$

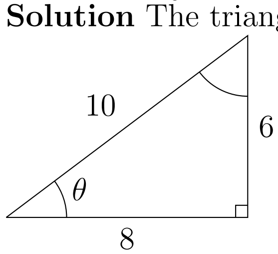
Solution The triangle we are looking at is



where the hypotenuse is given by the Pythagorean Teorem: $\sqrt{3^2 + 1^2} = \sqrt{10}$.
Then $\sec(\theta) = \frac{H}{A} = \frac{\sqrt{10}}{1} = \sqrt{10}$.

(b) $\tan(\arcsin(\frac{6}{10}))$

Solution The triangle we are looking at is



where the the other side length is given by the Pythagorean Teorem: $\sqrt{10^2 - 6^2} = \sqrt{64} = 8$.
Then $\tan(\theta) = \frac{6}{8}$.

(3) Do the following sequences converge? If so, to what?

(a) $a_n = \ln(3n + 1) - \ln(2n)$

Solution Yes, it converges to $\lim_{n\to\infty} \ln(3n + 1) - \ln(2n) = \lim_{n\to\infty} \ln \frac{3n + 1}{2n} = \ln \frac{3}{2}$

(b) $b_n = \arctan\left(\frac{n+1}{n}\right)$

Solution Yes, it converges to $\lim_{n\to\infty} \arctan\left(\frac{n+1}{n}\right) = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4}$

(c) $c_n = \sin(n\pi)$
Solution Yes, it converges to 0, since the sequence is the constant sequence $\{0, 0, 0, \dots\}$.

(4) Find the following limits:

(a) $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1}$
Solution Plugging in $x = 1$ only gives us a limit of the form $\frac{0}{0}$, so instead we rationalize the numerator:

$$\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} = \lim_{x\to 1} \frac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)(\sqrt{x+3}+2)} =$$

$$\lim_{x\to 1} \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)} = \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{x+3}+2)} = \lim_{x\to 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{\sqrt{1+3}+2} = \frac{1}{4}$$

(b) $\lim_{x\to 3} \sin(\pi x) - \cos(\frac{\pi}{2}(x-1))$
Solution The functions are continuous, so

$$\lim_{x\to 3} \sin(\pi x) - \cos(\frac{\pi}{2}(x-1)) = \sin(3\pi) - \cos(\frac{\pi}{2}(3-1)) = 0 - \cos(\pi) = 1.$$

(5) Using the Squeeze Theorem, find

$$\lim_{x\to 1} (x-1)^6 \cos\left(\frac{2x^2+1}{x-1}\right).$$

Solution

$$-1 \le \cos\left(\frac{2x^2+1}{x-1}\right) \le 1$$

$$-(x-1)^6 \le (x-1)^6 \cos\left(\frac{2x^2+1}{x-1}\right) \le (x-1)^6$$

Then $\lim_{x\to 1} -(x-1)^6 = 0$, $\lim_{x\to 1} (x-1)^6 = 0$, so by the Squeeze Theorem,

$$\lim_{x\to 1} (x-1)^6 \cos\left(\frac{2x^2+1}{x-1}\right) = 0$$