MATH1 Day 15: Continuity

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October 14, 2016

Continuity at a point

Definition

A function f is **continuous at a number** *a if*

$$\lim_{x \to a} f(x) = f(a).$$

Note! *If the function is not defined on one side of the point, we take $\lim_{x \to a} f(x)$ to be equal to the one-sided limit where the function is defined.*

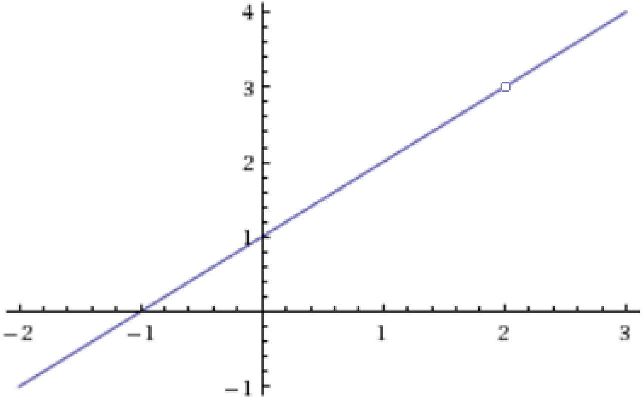
Continuity on an interval

Definition

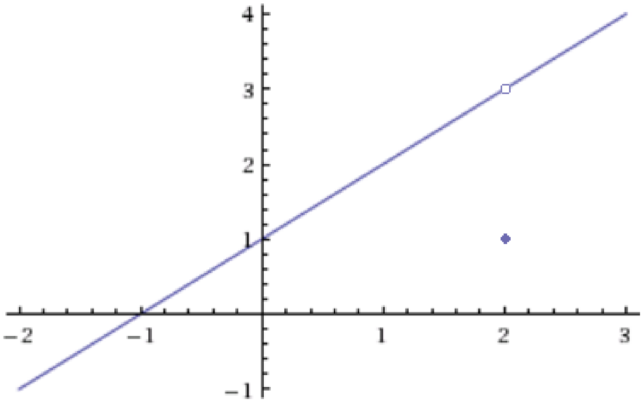
A function f is continuous on an interval if it is continuous at every number in the interval.

Using the definition of continuity, answer the following question: Is $f(x)$ continuous at $x = 2$? Is $f(x)$ continuous on its domain?

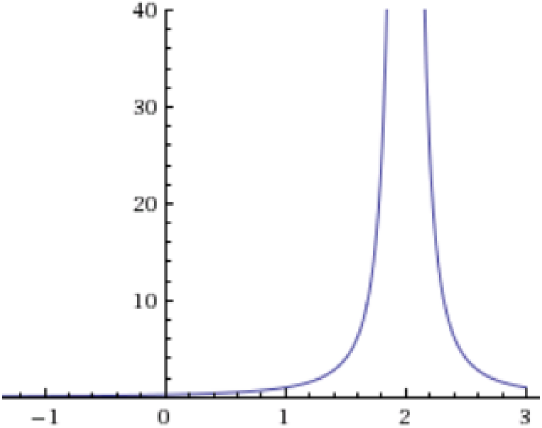
$f(x) = \frac{x^2-x-2}{x-2}$



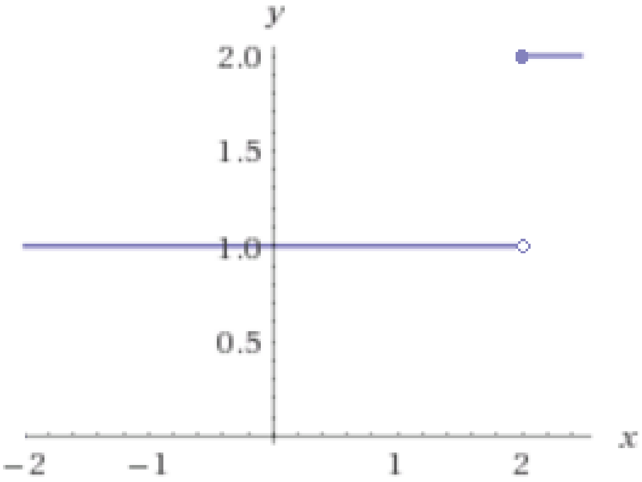
$$f(x) = \begin{cases} 1 & \text{if } x = 2 \\ \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \end{cases}$$



$f(x) = \frac{1}{(x-2)^2}$



$$f(x) = \begin{cases} 1 & \text{if } x < 2 \\ 2 & \text{if } x \geq 2 \end{cases}$$



Continuity properties

Theorem

If f and g are continuous at a and if c is a constant, then the following functions are also continuous at a:

1. $f + g$
2. fg
3. $f - g$
4. $\frac{f}{g}$ if $g(a) \neq 0$
5. cf

Continuity properties - part II

Theorem

If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

Intermediate Value Theorem

Theorem

Suppose that f is continuous on the closed interval $[a, b]$ and let S be any number between $f(a)$ and $f(b)$ where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = S$.

The Comparison Theorem

Theorem

If $f(x) \leq g(x)$ *when* x *is near* a *(except possibly at* a*), and both* $\lim_{x\to a} f(x)$ *and* $\lim_{x\to a} g(x)$ *exist, then*

$$\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x).$$

The Squeeze Theorem

Theorem

If $f(x) \le g(x) \le h(x)$ *when* x *is near* a *(except possibly at* a*), and*

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L,$$

then

$$\lim_{x \to a} g(x) = L.$$