Final Exam
Math 1
December 3, 2011

Name: _____

Please circle your instructor's name below:

Harris LaLonde

Please read the following instructions before starting the exam:

- This exam is closed-book, with no calculators, notes, or books allowed. You may not give or receive any help on the exam, though you may ask the instructors for clarification if necessary.
- Be sure to show all your work wherever possible. Even if your final answer is incorrect, we can assign an appropriate amount of partial credit if we can see how you arrived at your answer.
- Please circle or otherwise indicate your final answer.
- This test has a total of 12 questions, worth a total of 150 points. Point values are indicated for each question.
- You will have three hours from the start of the exam to complete it.
- Good luck!

Honor statement: I have neither given nor received any help on this exam, and I attest that all of the answers are my own work.

Signature: _____

This page is for grading purposes only.

Problem	Score	Points
1		12
2		12
3		8
4		6
5		10
6		25
7		12
8		10
9		12
10		8
11		25
12		10
Total		150

1. [12 points] **Multiple choice.** Circle the correct answer for each question. Each part is worth 2 points.

(a) The inverse function of 2^x is

A. $\left(\frac{1}{2}\right)^x$

B. $\log_2(x)$

C. 2^{-x}

D. The function does not have an inverse.

(b) The domain of $\sin^{-1}(x)$ is

A. $[-1, 1]$

B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

C. $[-\pi, \pi]$

D. $(-\infty, \infty)$

(c) The function $f(x) = e^{-x}$ is concave down

A. everywhere.

B. on $(-\infty, 0)$.

C. on $(0, \infty)$.

D. nowhere.

(d) Which of the following statements is always true?

A. If $f'(c) = 0$, then $(c, f(c))$ is a local maximum or a local minimum of $f(x)$.

B. If $(c, f(c))$ is a local maximum or a local minimum of $f(x)$, then $f'(c) = 0$.

C. If $f'(c) = 0$ and $f''(c) < 0$, then $(c, f(c))$ is a local maximum of $f(x)$.

D. None of the above statements are always true.

(e) The function $g(x) = \dfrac{1}{1 - e^x}$ has

A. one vertical asymptote and one horizontal asymptote.

B. one vertical asymptote and two horizontal asymptotes.

C. two vertical asymptotes and one horizontal asymptote.

D. two vertical asymptotes and two horizontal asymptotes.

(f) What does

$$\lim_{h \to 0} \frac{\ln(2 + h) - \ln(2)}{h}$$

equal? [Hint: What does this formula represent?]

A. $\ln 2$

B. $-\ln 2$

C. $\dfrac{1}{2}$

D. $-\dfrac{1}{4}$

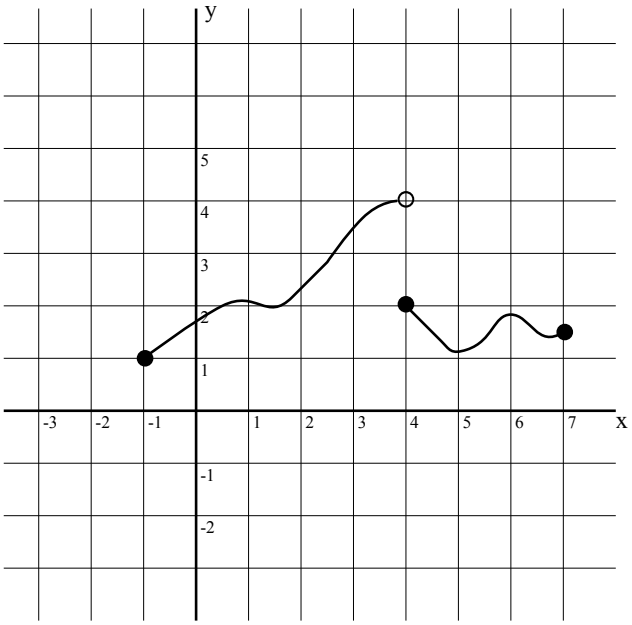
2. [12 points] **Short answer.** Please answer as clearly and precisely as possible. Each question is worth 3 points.

(a) Complete the following statement: "The Chain Rule states that $(f \circ g)'(x)$ is equal to ..."

(b) Suppose that $D(t)$ measures the deer population in Hanover t years after 2000. If I told you that $D'(11) = -200$, explain in a sentence how you would interpret that.

(c) Draw a differentiable function with one local maximum, one inflection point, and no other critical points or inflection points.

(d) Below is the graph of a function $f(x)$ defined on the closed interval $[-1, 7]$.



Observe that f doesn't have an absolute maximum on $[-1, 7]$. Explain why this does not violate the Extreme Value Theorem.

3. [8 points] Let $h(x) = \ln(1 + 4e^x)$.

(a) [4 points] What is $h^{-1}(x)$?

(b) [4 points] What should $(h \circ h^{-1})(x)$ be? Compute $(h \circ h^{-1})(x)$ to verify that this is true.

4. [6 points] What value of k will make $f(x)$ continuous everywhere?

$$f(x) = \begin{cases} \dfrac{2x^2 + x - 10}{x - 2} & x < 2 \\ \\ k\cos(\pi x) & x \geq 2 \end{cases}$$

5. [10 points] Find each of the following limits.

(a) [4 points]

$$\lim_{x \to 1} \frac{x-1}{x^2+2x-3}$$

(b) [4 points]

$$\lim_{x \to -3^+} \frac{x-1}{x^2+2x-3}$$

(c) [2 points] The function $f(x) = (x-1)/(x^2+2x-3)$ is not continuous at $x = 1$ or $x = -3$. Based on what you found in (a) and (b), what type of discontinuity does f have at each of these points?

6. [25 points] Find the derivative of each of the following functions.

(a) [5 points]

$$f(x) = (3x + 7)^4(2x^4 + 7x^2 + 1)^8$$

(b) [5 points]

$$F(t) = \frac{3e^t}{\sqrt{2t + 1}}$$

(c) [5 points]

$$G(x) = \sin^2(\ln(x))$$

(d) [5 points]

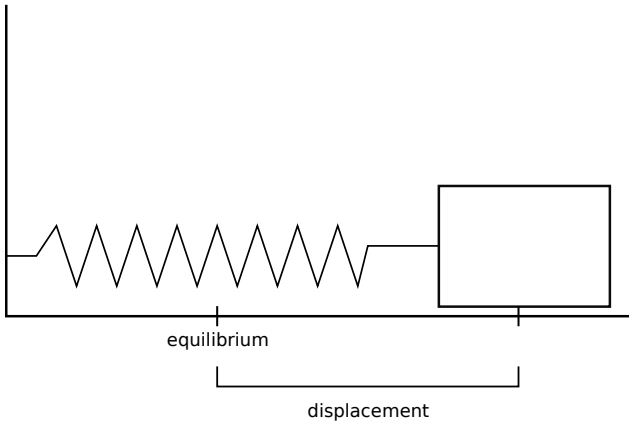
$$y = e^{x^2 \tan(x)}$$

(e) [5 points]

$$h(t) = \arctan(\sqrt{t})$$

7. [12 points] A block is attached to a spring and set in motion, which causes it to slide back and forth along a surface. The displacement (in centimeters) from the block's equilibrium position at t seconds is given by the function

$$f(t) = 4e^{-t}\sin(t).$$



(a) [4 points] Find $\lim_{t\to\infty} f(t)$.

(b) [2 points] Interpret your answer to part (a) physically.

(c) [6 points] Find the velocity and acceleration of the block as functions of time. Include proper units.

8. [10 points] Consider the implicit equation

$$7x^2 + 2xy + e^y = 8.$$

(a) [6 points] Find y'.

(b) [4 points] Find the equation of the tangent line to this curve at the point $(-1, 0)$.

9. [12 points] Use logarithmic differentiation to find y' in each of the following cases.

(a) [6 points]

$$y = \frac{7x^3 e^{x^2} \sin(x)}{x^5 + 2x^2 + 1}$$

(b) [6 points]

$$y = x^{\ln(x)}$$

10. [8 points] Find the absolute maximum and absolute minimum of the function

$$f(x) = \frac{x}{x^2 - x + 1}$$

on the interval $[0, 2]$.

11. [25 points] Consider the function

$$f(x) = 3x^4 - 4x^3.$$

(a) [3 points] Find the x- and y-intercepts of $f(x)$.

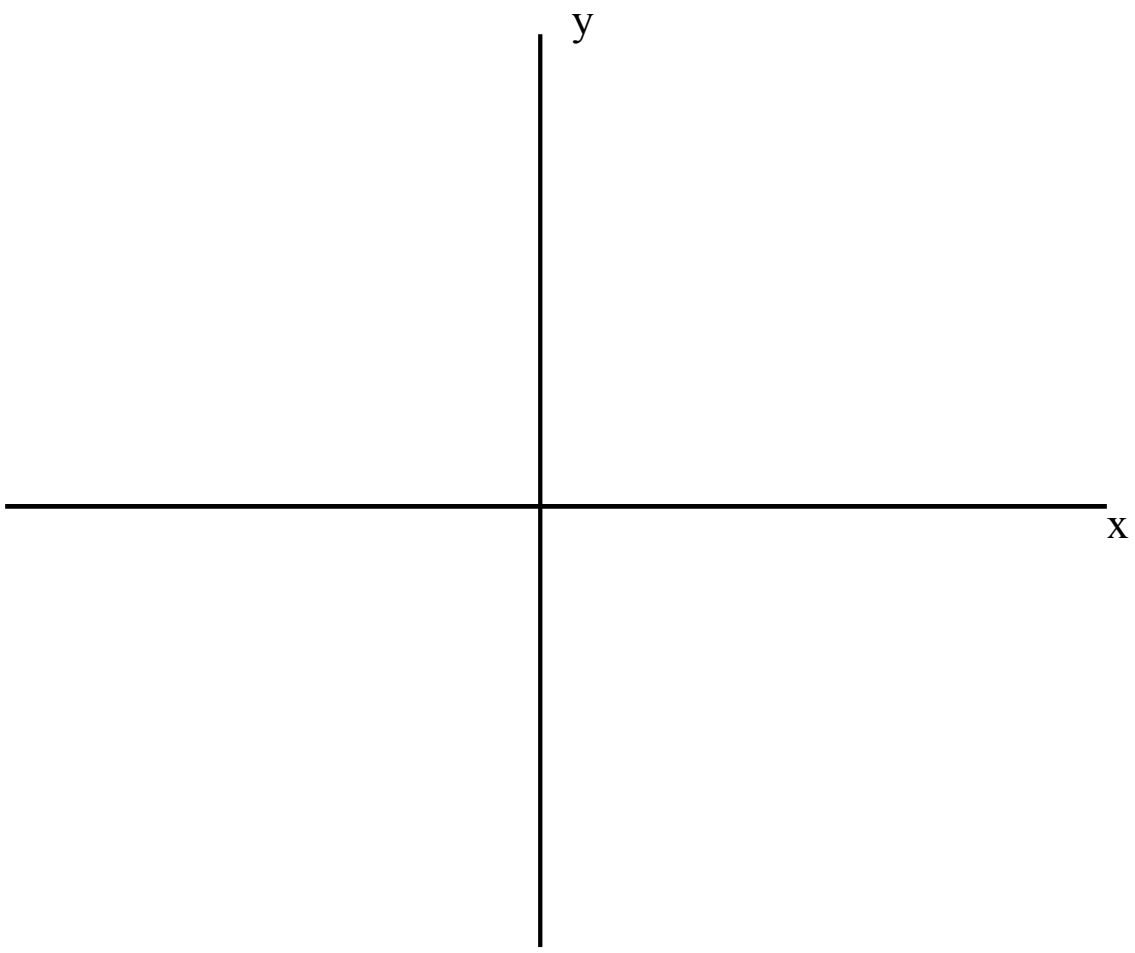
(b) [6 points] Find the critical points of f and the value of f at each point. Classify each as a local maximum, local minimum, or neither.

(c) [3 points] Determine the intervals on which f is increasing and decreasing.

(d) [4 points] Find the inflection points of f and the value of f at each point.

(e) [3 points] Determine the intervals on which f is concave up or concave down.

(f) [6 points] Sketch the graph of f. Label all critical points, inflection points, and intercepts. (Some values may not be integers - give your best estimate for these values.)



12. [10 points] The power P (measured in watts) dissipated by a resistor with V volts applied across it is given by the formula

$$P = \frac{V^2}{R},$$

where R is the resistance (measured in ohms). At a certain instant in time, V is 5 volts, R is 10 ohms, and V and R are changing at rates of 6 volts/min and 4 ohms/min, respectively. Find the rate of change of P at this instant. Include proper units.