Practice Final Exam

Math 1

November 29, 2011

Name: ______________________________

Here are some notes regarding this practice exam:

- This exam is in no way complete - it is simply meant to be an indicator of the sorts of problems you may see on the actual exam. The real exam will be longer, and not all of the concept that we have covered are represented on this practice test.
- If you want to simulate the real exam experience, you should try to do the problems first without looking at your notes or book, then refer to them later on to fill in any things you may have missed. You should also try timing yourself; this exam is roughly two-thirds of the length of the real thing, so you should try to do it in about two hours.
- After you've attempted the exam, you may come talk to me about how to do the problems.
- To reiterate: this is **not** intended to be a guide for what to study. You should be familiar with *all* the topics listed on the "Notes on the Final Exam" sheet that has been posted alongside this. You should expect to be tested on any of those topics, even though they may not appear on this practice test.

1. Short answer.

(a) Explain what is wrong with the following statement: "Every function $f(x)$ attains an absolute maximum and an absolute minimum on a closed interval $[a, b]$."

(b) Explain what is wrong with the following statement: "If $f(x)$ has a local maximum at $(c, f(c))$, then $f'(c) = 0$."

(c) What type of function passes the Horizontal Line Test? What important property do these functions possess?

2. Let $f(x) = \dfrac{x^2 - 1}{x^2 - x}$.

(a) Find $\lim_{x \to 1} f(x)$.

(b) Find $\lim_{x \to 0^-} f(x)$.

(c) Find $\lim_{x \to +\infty} f(x)$.

(d) What is the domain of $f(x)$?

3. What value of k will make $f(x)$ continuous everywhere?

$$f(x) = \begin{cases} \dfrac{\sqrt{25-x} - 5}{x} & x < 0 \\[2mm] \dfrac{x-1}{k} & x \geq 0 \end{cases}$$

4. Use the limit definition of the derivative to find $f'(x)$, where $f(x) = x^3 - 4x + 6$.

5. Find the derivatives of the following functions:

(a) $f(x) = x(1 + 2x + 3x^2)^{10}$

(b) $g(x) = e^x + x^e + e^e$

(c) $h(x) = \dfrac{1}{\sqrt[3]{\cos(x) + \arctan x}}$

(d) $j(x) = \tan^2(e^{2x} + e^{-2x})$

(e) $k(x) = \sqrt{x}^{\sqrt{x}}$ (Use logarithmic differentiation.)

6. Find the maximum and minimum values of $f(x) = x + \dfrac{4}{x}$ on the interval $[1, 4]$.

7. Consider the function $f(x) = -x^4 + 72x^2 + 100$.

(a) Find the critical points of $f(x)$ and classify them as local minima or local maxima.

(b) Find the intervals on which $f(x)$ is increasing and decreasing.

(c) Find the points of inflection of $f(x)$.

(d) Find the intervals on which $f(x)$ is concave up and concave down.

(e) Sketch the graph of $f(x)$. Label the critical points, inflection points, and y-intercept. (Some values may not be integers; give your best estimate for these values.)

8 Let $g(x) = e^{-4x} + 8x$.

(a) $g(x)$ only has one critical point. What is it?

(b) Use the Second Derivative Test to show whether this critical point is a local maximum, a local minimum, or neither.

9. A ladder 41 feet long that was leaning against a wall begins to slip. Its top slides down the wall while its bottom moves along the ground at a constant speed of 4 ft/sec. How fast is the top of the ladder moving when it is 9 feet above the ground?

10. Find an equation to the line tangent to the curve $2e^{-x} + e^{y} = 3e^{x-y}$ at the point (0,0).