Math 1: Calculus with Algebra
Midterm 1
Thursday, October 8

Name: _____

Circle your section number: 1–Freund   2–DeFord

Please read the following instructions before starting the exam:

- This exam is closed book, with no calculators, notes, or books allowed. You may not give or receive any help during the exam, though you may ask instructors for clarification if necessary.

- Be sure to **show all work** whenever possible. Even if your final answer is incorrect, we can assign an appropriate amount of partial credit if we can see how you arrived at your answer.

- Please circle or otherwise indicate your final answer if possible.

- The test has a total of 12 questions, worth a total of 120 points. Point values are indicated for each question.

- You will have two hours from the start of the exam to complete it.

- Good luck!

HONOR STATEMENT: I have neither given nor received help on this exam, and I attest that all the answers are my own work.

Signature: _____

This page for grading purposes only.

Problem	Points	Scores
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
10	12	
Total	120	

1. **(12 points)** Throughout this problem, let $f(x) = 2x^2 - x + 3$.

   (a)(5 pts) Find the average rate of change of f over the interval $[-1, 3]$.

   (b)(5 pts) Find the average rate of change of f over the interval $[2, 5]$.

   (c)(2 pts) What is the geometric interpretation of the average rate of change over an interval?

2. **(12 points)** *Function Adjectives.*

(a)(6 pts) Consider the following graph of $f(x)$ and fill in the information below:
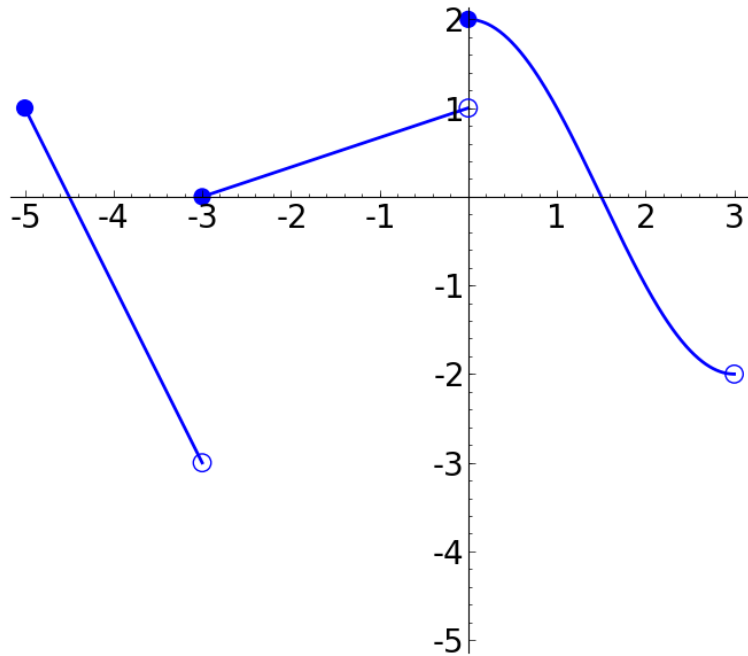
I) The domain of f is: _____

II) The range of f is: _____

III) f is increasing on: _____

IV) f is decreasing on: _____

(b)(6 pts) Consider the functions $g(x) = 2x^2 - \sqrt{x} + 1$ and $h(x) = x^2 - 3x + 2$.

I) What is the domain of g?

II) What is the domain of $\left(\frac{g}{h}\right)$?

III) Write the formula for the function $(h - g)(x)$.

3. **(12 pts)** *Function Composition.*

(a)(4 pts) Let $f(x) = \dfrac{2}{x-1}$ and $g(x) = \dfrac{2+x}{x}$. Find and simplify $(f \circ g)(x)$.

(b)(4 pts) Write $h(x) = (x+2)^2$ as a composition of two functions. Show that your decomposition works.

(c)(4 pts) Write $g(x) = \cos(\ln(x)) + 5\ln(x)$ as a composition of two functions. Show that your decomposition works.

4. **(12 pts)** *Graph Transformations.*

(a)(6 pts) Write the transformations, in order, that are needed to obtain the graph of $g(x) = -\sqrt{x} + 2$ from the graph of $f(x) = \sqrt{x}$.

(b)(6 pts) Write the algebraic expression that will transform the graph of the function $h(x) = e^x$ by compressing horizontally by a factor of 2, reflecting across the x–axis, and *then* shifting the graph down by $\frac{1}{3}$.

5. **(12 points)** *Sequences.*

(a)(6 pts) Consider the sequence $\{2n + 5\}_{n=0}^{\infty}$.

I) Write out the first 5 terms of this sequence.

II) Determine whether the sequence is bounded. Explain your conclusion (1-2 sentences).

(b)(6 pts) Consider the sequence $\{-3, 6, -9, 12, -15, \ldots\}$.

I) Find a closed form for this sequence.

II) Determine whether this sequence is increasing, decreasing, or neither.

6. (**12 pts**) *Inverse Functions.*

(**a**)(**3 pts**) Find the inverse of the function $f(x) = \dfrac{4}{2x+1}$.

(**b**)(**3 pts**) Compute the domain and range of $f^{-1}(x)$.

(**c**)(**3 pts**) Is the function $h(x) = \cos(x)$ one–to–one? If so, explain why. If not find points a and b so that $h(a) = h(b)$.

(**d**)(**3 pts**) Is the function $j(x) = 3x^3 - 4$ one–to–one? If so, explain why. If not find points a and b so that $j(a) = j(b)$.

7. **(12 pts)** *Simplifying Expressions.*

(a)(4 pts) Simplify the expression: $\left(\dfrac{x^{\frac{1}{2}}z^4y}{y^2}\right)^{-2}$

(b)(4 pts) Solve $3e^{4x+1} = 24$ for x.

(c)(4 pts) Solve $\log_2((x+2)^3) = -6$ for x.

8. **(12 pts)** *Trigonometry.*

(a)(6 pts) Write an algebraic expression for $\cos(\arctan(x))$

(b)(6 pts) If $\cot(\theta) = \frac{2}{3}$, find the other 5 trigonometric ratios for θ.

9. **(12 points)** *Multiple Choice.* Please circle your answers.

(a)(2 pts) What is the value of $\log_2(\frac{1}{8})$?

   I) $\frac{1}{3}$

   II) -3

   III) $-\frac{1}{3}$

   IV) 3

(b)(2 pts) If $f(x) = x^2 - 2x + 1$, which function represents $f(3x)$?

   I) $3x^2 - 6x + 1$

   II) $3x^2 - 6x + 3$

   III) $9x^2 - 6x + 1$

   IV) $9x^2 - 6x + 3$

(c)(2 pts) What is the domain of $\log_4(x)$?

   I) $(0, \infty)$

   II) $(-\infty, \infty)$

   III) $[0, \infty)$

   IV) $(-\infty, 0) \cup (0, \infty)$

(d)(2 pts) What is $\sin(\arccos(1))$?

I) $\frac{1}{\sqrt{2}}$

II) -1

III) 1

IV) 0

(e)(2 pts) What is the x–intercept of the function $x^2 + 4x + 4$?

I) 2

II) -2

III) 0

IV) 4

(f)(2 pts) What is the y–intercept of the function below?

$$e^{12x} - (\ln(x+1) + \tan(x))$$

I) 2

II) 0

III) -1

IV) 1

10. **(12 points)** *True or False.* If the statement is true, explain why. If the answer is false, explain why or give an example that disproves the statement.

(a)(3 pts) _____ If f is a function then $f(2x - 1) = 2f(x - 1)$.

(b)(3 pts) _____ The function $h(x) = x^2 + 3x + 1$ is an even function.

(c)(3 pts) _____ For any two functions f and g, $f \circ g = g \circ f$.

(d)(3 pts) _____ If f is an increasing function, then $f(x + 1) > f(x)$ for all x.