Math 1: Calculus with Algebra
Midterm 2
Thursday, October 29

Name: Answer Key

Circle your section number: 1–Freund 2–DeFord

Please read the following instructions before starting the exam:

- This exam is closed book, with no calculators, notes, or books allowed. You may not give or receive any help during the exam, though you may ask instructors for clarification if necessary.
- Be sure to **show all work** whenever possible. Even if your final answer is incorrect, we can assign an appropriate amount of partial credit if we can see how you arrived at your answer.
- Please circle or otherwise indicate your final answer if possible.
- The test has a total of 10 questions, worth a total of 120 points. Point values are indicated for each question.
- You will have two hours from the start of the exam to complete it.
- Good luck!

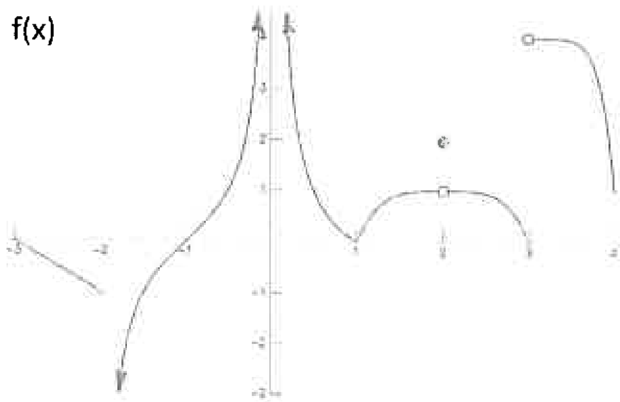
HONOR STATEMENT: I have neither given nor received help on this exam, and I attest that all the answers are my own work.

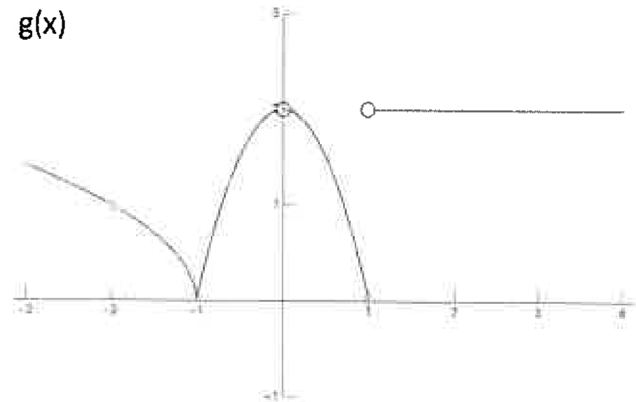
Signature: ______________________________

This page for grading purposes only.

Problem	Points	Scores
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
10	12	
Total	120	

1. **(12 points)** Use the graphs of f and g to compute the stated limits.





(a) **[3 pts]** $\lim_{x \to 0} f(x)$ ∞

(b) **[3 pts]** $\lim_{x \to 2} f(x)$ 1

(c) **[3 pts]** $\lim_{x \to 1} (f(x) + g(x))$ $(0 + \text{DNE}) = \text{DNE}$

(d) **[3 pts]** $\lim_{x \to 2} \frac{f(x)}{g(x)}$ $\frac{1}{2}$

2. **(12 points)** Use the functions below to find the stated limits.

$$f(x) = \frac{x^2 - 16}{x - 4} \qquad g(x) = \begin{cases} x^2 - 3 & x < -2 \\ x^3 + 9 & -2 \le x < -1 \\ x + 2 & -1 \le x \end{cases} \qquad h(x) = \frac{1}{x - 2}$$

(a) **[2 pts]** $\lim_{x \to 4} f(x)$ 8

(b) **[2 pts]** $\lim_{x \to -4} g(x)$ 13

(c) **[2 pts]** $\lim_{x \to -1^+} g(x)$ 1

(d) **[2 pts]** $\lim_{x \to -1^-} g(x)$ 8

(e) **[2 pts]** $\lim_{x \to 2^+} h(x)$ ∞

(f) **[2 pts]** $\lim_{x \to 2^-} h(x)$ $-\infty$

3. **(12 points)** Throughout this problem, let $f(x) = x^2 + 8x^{-2} - 4$.

(a) [**4 pts**] Find the slope of tangent line of f at $x = 2$.

$f'(x) = 2x - 16x^{-3}$

$f'(2) = 4 - \frac{16}{8} = 2$

(b) [**4 pts**] Find the equation for the tangent line of f at $x = 2$.

$y - 2 = 2(x - 2)$

(c) [**4 pts**] Find the instantaneous rate of change of f at $x = 1$.

$f'(1) = -14$

4. **(12 points)** In the following problem, you may draw graphs to *support* your answer. However, they should not be your only source of justification.

(a) [**4 pts**] Determine whether $h(x) = \begin{cases} x^3 - 5 & x < 0 \\ 5\cos(x) & 0 \leq x \end{cases}$ is continuous at $x = 1$.

5 cos(x) is continuous at 1.

(b) [**4 pts**] Determine whether $s(x) = \begin{cases} \frac{1}{x} & x < 2 \\ x^2 - 2x & x \geq 2 \end{cases}$ is continuous at $x = 2$.

$\lim_{x \to 2^-} s(x) = \frac{1}{2}$

$\lim_{x \to 2^+} s(x) = 0$

not equal so not continuous

(c) [**4 pts**] For what values of x is $r(x) = \dfrac{x^2 + 3x - 10}{x^2 - x - 2}$ continuous? Explain your conclusion in 1-2 sentences.

$(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

Rational functions are continuous on their domains, and the roots of the denominator are -1 and 2.

5. **(12 points)** For parts (a) and (b), justify your conclusions completely.

(a) [**4 pts**] Consider $h(x) = x^2 - 1$ over the interval $[0, 1]$. Write a value y such that there must exist a c between 0 and 1 such that $h(c) = y$.

Any $y \in [-1, 0]$ by IVT.

(b) [**4 pts**] Show that $f(x) = x^3 + 4x^2 - 3$ has a zero between 0 and 2.

$f(0) = -3$

$-3 \leq 0 \leq 21$

$f(2) = 21$

By IVT there exists a $0 \leq c \leq 2$ with $f(c) = 0$.

(c) [**4 pts**] Consider $g(x) = \begin{cases} 3x + 4 & x < 2 \\ 2x - 1 & x \geq 2 \end{cases}$. On which of the following intervals (if any) does the Intermediate Value Theorem apply? (YES or NO)

I. $(-\infty, \infty)$ NO

II. $[4, 20]$ Yes

III. $[-3, 0)$ NO

IV. $(-10, 10]$ NO

6. **(12 points)** *(Discontinuities)*

(a) [**6 pts**] List the discontinuities of $f(x) = \begin{cases} \dfrac{x^2 - 2x - 8}{x+2} & x \leq 0 \\ \frac{-12}{x} & x > 0 \end{cases}$. For each one, determine the type of discontinuity.

$x = -2$ Removable

$x = 0$ infinite

(b) [**4 pts**] What does it mean to say that a discontinuity is a "jump" discontinuity? You may give an intuitive description but also include a precise definition.

$$\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$$

(c) [**2 pts**] Create a function with a removable discontinuity at $x = 3$.

$$\frac{x-3}{x-3}$$

7. **(12 points)** *(Derivative Definition)*

(a) **[2 pts]** State one of the limit definitions of the derivative of f at a point a.

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{or} \quad \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

(b) **[2 pts]** State one of the limit definitions for the derivative of $f(x)$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(c) **[8 pts]** Find the derivative of $f(x) = \sqrt{x+1}$ using the limit definition.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{a}$$

Rationalize numerator

$$= \lim_{h \to 0} \frac{x+h+1 - x - 1}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \frac{1}{2\sqrt{x+1}}$$

8. **(12 pts)** *(Sequences)*

(a) **[2 pts]** Write the formula for a bounded sequence.

$$\left\{ \frac{1}{n} \right\}_{n=1}^{\infty}$$

(b) **[2 pts]** Write a formula for a sequence that doesn't converge.

$$\left\{ n \right\}_{n=1}^{\infty}$$

(c) **[8 pts]** Prove that $\left\{ \frac{n^2-2}{3n^2+3} \right\}_{n=1}^{\infty}$ converges to $\frac{1}{3}$.

$|a_n - L| < \varepsilon$

$$\left| \frac{n^2-2}{3n^2+3} - \frac{1}{3} \right| < \varepsilon$$

$$\left| \frac{n^2-2}{3n^2+3} - \frac{n^2+1}{3n^2+3} \right| < \varepsilon$$

$$\left| \frac{-3}{3n^2+3} \right| < \varepsilon$$

$$\frac{1}{n^2+1} < \varepsilon$$

$$\frac{1}{\varepsilon} < n^2+1$$

$$\sqrt{\frac{1}{\varepsilon} - 1} < n$$

9. **(12 points)** Compute the derivatives for each of the following by any means:

(a) **[3 pts]** $4^2 - 3x + x^3$

$3x^2 - 3$

(b) **[3 pts]** $\dfrac{x^2 + 4x + 5}{\sqrt{x}}$ $= x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + 5x^{-\frac{1}{2}}$

$\frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} - \frac{5}{2}x^{-\frac{3}{2}}$

(c) **[3 pts]** $\dfrac{2}{x^2} - 2x^2 + \pi x^{\pi}$

$\dfrac{-4}{x^3} - 4x + \pi^2 x^{\pi - 1}$

(d) **[3 pts]** $x^e + e^x - 17$

$ex^{e-1} + e^x$

10. **(12 points)** *True or False.* If the statement is true, explain why. If the answer is false, explain why or give an example that disproves the statement.

(a) **[3 pts]** T If a function f is differentiable at a then f is continuous at a.

Theorem

(b) **[3 pts]** F A function g has a vertical asymptote at $x = a$ if $g'(a) = 0$.

$g'(a) = 0$ is a horizontal tangent line.

(c) **[3 pts]** F For *any* function h, $\lim_{x \to a} h(x) = h(a)$.

continuous

(d) **[3 pts]** T The function $s(x) = \cos(2x^3 - 5) + e^{7x}$ is continuous on $\mathbb{R}$.

Each function is continuous