MATH 1: INTRODUCTION TO CALCULUS
MIDTERM EXAM #2

Name: ______________________________

Section (please circle): 10 Winkeler   11 Tripp   12 Chen   2 Xiao

(1) Write your name *legibly* and circle your section above.
(2) This exam is closed book. You may not use notes, calculators, or any other external resource. It is a violation of the honor code to give or receive help on this exam.
(3) You must explain what you are doing, justify your answer, and show your work. You will be *graded on your work*, not just on your answer. Please write clearly.
(4) It is fine to leave your answer in a form such as $14\sqrt{239}$. However, if an expression can be easily simplified (such as $\cos(\pi/2)$ or $2+3$), please simplify it.
(5) If you use the backside of a page or the scratch paper at the end of the exam, and you want us to look at it, please write on a problem "Continued on back/page…".

Problem	**Score**	**Problem**	**Score**
1	/10	6	/6
2	/8	7	/12
3	/8	8	/8
4	/10		
5	/12		
		Total	/74

Date: Thursday, 31 October 2019.

Problem 1.[10 points] Mark the following statements as true or false.

_____ $f'(a)$ is the slope of the tangent line to $f(x)$ at a.

_____ $g(x) = \frac{1}{x}$ is continuous on its domain.

_____ $2, 4, 6, 8, 10, 12, \ldots$ is a geometric sequence.

_____ The sequence $a_n = \sin(n)$ is bounded.

_____ $\lim\limits_{x \to 0} \ln x = -\infty$.

_____ If $h(x) = 2x$, then $h'(x) = x^2$.

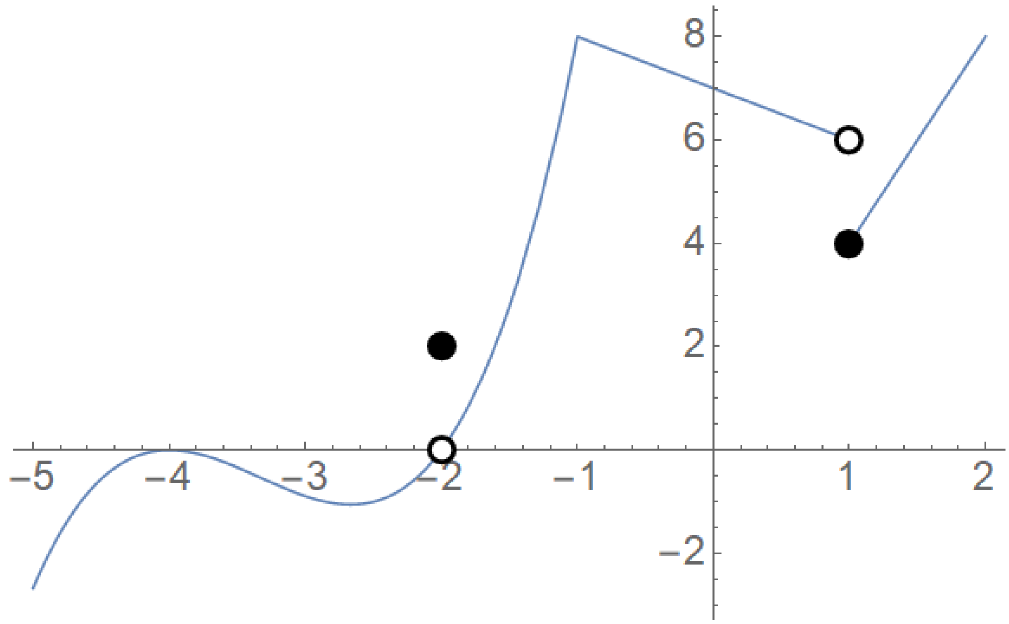
_____ If $j(x)$ is a continuous function such that $j(0) = 0$ and $j(1) = 1$, then $j(\frac{1}{2}) = \frac{1}{2}$.

_____ A monotonic sequence is either bounded above or bounded below.

_____ The value of a continuous function cannot approach ∞ as $x \to \infty$.

_____ The derivative of $g(x) = |x|$ at $x = 0$ is 0.

Problem 2. [8 points] Use the graph of $y = f(x)$ below to answer the following questions.



(a) Find the following limits. Write DNE if the limit does not exist.

(i) $\lim_{x \to -4^+} f(x) =$ __________

(ii) $\lim_{x \to -2} f(x) =$ __________

(iii) $\lim_{x \to -1} f(x) =$ __________

(iv) $\lim_{x \to 1^-} f(x) =$ __________

(v) $\lim_{x \to 1} f(x) =$ __________

(b) At what values of x is $f(x)$ not differentiable?

Problem 3. [8 points]

(a) Use the Squeeze Theorem to find $\lim_{x\to 0} x^2 \sin(\ln|x|)$

(b) Use the Intermediate Value Theorem to show that $f(x) = x + 2^x$ has a zero.

Problem 4. [10 points]

(a) State the three requirements for $f(x)$ to be continuous at a point a.

(b) List the three types of discontinuities a function can have.

(c) Find and classify all discontinuities of $f(x) = \dfrac{x+1}{x^2-1}$.

Problem 5. [12 points] Consider the piecewise function

$$f(x) = \begin{cases} x+2, & x < 2 \\ a, & x = 2 \\ \frac{c(x^2-b^2)}{x-2}, & x > 2 \end{cases}.$$

where a, b and c are some constants.

(a) Let $c = 1$. Solve for a and b so that this function is continuous everywhere.

(b) Let $a = 4$, $b = 3$, $c = 1$, so we get

$$f(x) = \begin{cases} x+2, & x < 2 \\ 4, & x = 2 \\ \frac{(x^2-9)}{x-2}, & x > 2 \end{cases}.$$

Is $f(x)$ continuous at $x = 2$? If not, what type of discontinuity does it have?

(c) Let $a = 3$, $b = 2$, $c = 1$, so we get

$$f(x) = \begin{cases} x+2, & x < 2 \\ 3, & x = 2 \\ \frac{(x^2-4)}{x-2}, & x > 2 \end{cases}.$$

Is $f(x)$ continuous at $x = 2$? If not, what type of discontinuity does it have?

(d) Let $a = 1$, $b = 2$, $c = 3$, so we get

$$f(x) = \begin{cases} x+2, & x < 2 \\ 1, & x = 2 \\ \frac{3(x^2-4)}{x-2}, & x > 2 \end{cases}.$$

Is $f(x)$ continuous at $x = 2$? If not, what type of discontinuity does it have?

Problem 6. [6 points]

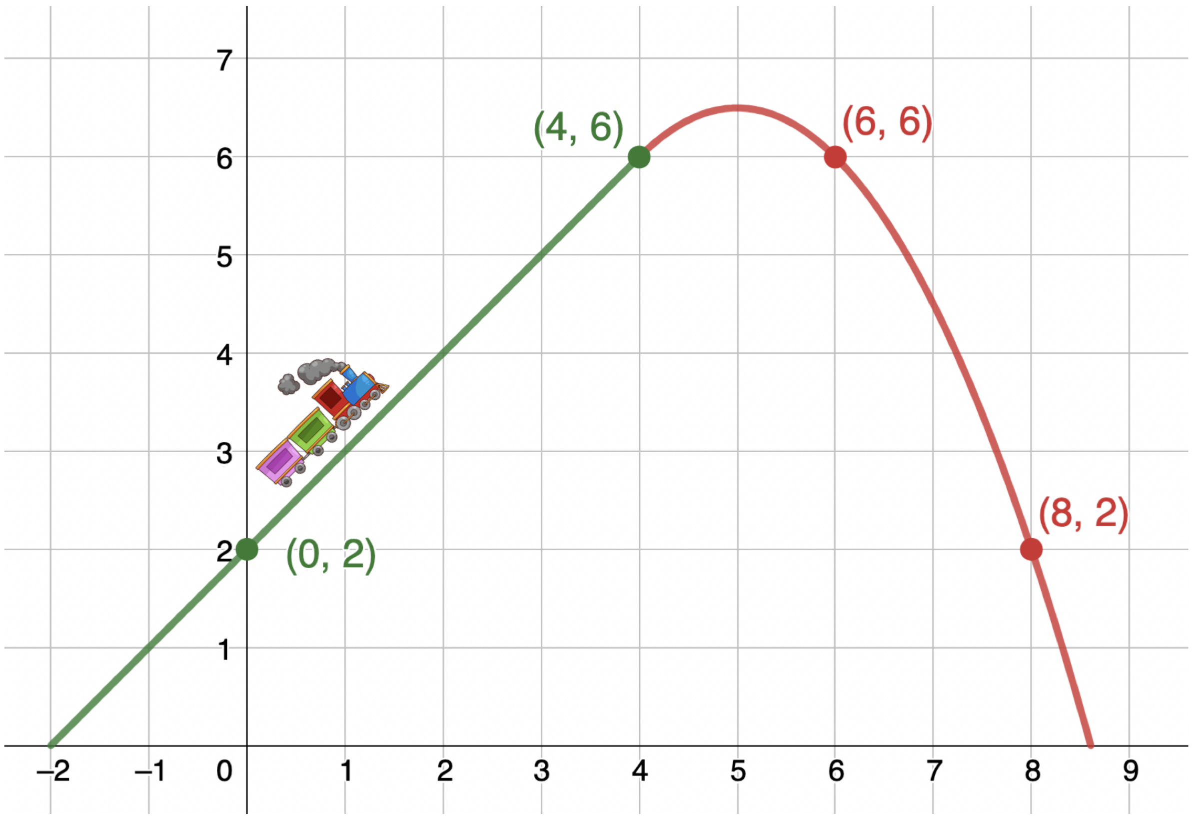
(a) Suppose $\lim_{x\to 2} f(x) = 3$. Use the algebraic limit laws to compute $\lim_{x\to 2} \sqrt{10 - (f(x))^2}$.

(b) Suppose $\lim_{x\to 3} g(x) = 4$ and $\lim_{x\to 3} h(x) = 1$. Use the algebraic limit laws to compute $\lim_{x\to 3} \frac{4g(x)}{1+h(x)}$.

(c) Suppose $\lim_{x\to 2^-} s(x) = -3$ and $\lim_{x\to 2^+} s(x) = 3$. Use the algebraic limit laws to compute $\lim_{x\to 2} s(x)$ and $\lim_{x\to 2}(s(x)^2)$.

Problem 7. [12 points] As you may still remember from the First Midterm, the figure below shows a track of the Green-Red Mountain. Suppose the figure depicts the entire mountain, modeled by the function:

$$f(x) = \begin{cases} x+2, & x \leq 4 \\ -\frac{1}{2}x^2 + 5x - 6 & x > 4 \end{cases}$$



(a) Let $f(x)$ be a function. State the limit definition of the derivative function $f'(x)$.

(b) Compute the derivative of $x+2$ at $x = 4$ and the derivative of $-\frac{1}{2}x^2+5x-6$ at $x = 4$ using the limit definition. Verify that they are the same, and the track smoothly transitions from the green to the red. [Hint: compute the derivative function and plug in, as the derivative function will be used again in the next part.]

(c) Engineers are working to decide if it is safe for a train to travel on the mountain or if they need to build a tunnel through the mountain. Compute the slope of the tangent line to the mountain at the foot of the mountain, the point $(8, 2)$, using the limit definition. If the absolute value of the slope is greater than 2, then it is too dangerous and the engineers need to build a tunnel. Do they need to build a tunnel? [Work must be shown to receive credit on this problem.]

☺ **Problem 8**. [8 points]

(a) For each of the following sequences, state whether they are arithmetic, geometric, neither, or both. Write down an **explicit** formula for each.

(i) $\{a_n\} = \{3, 7, 11, 15, 19, 23, 27, \ldots\}$

(ii) $\{b_n\} = \{2, \frac{3}{7}, \frac{4}{49}, \frac{5}{343}, \frac{6}{2401}, \ldots\}$ (hint: $7 = 7^1$, $49 = 7^2$, $343 = 7^3$, $2401 = 7^4$)

(b) Suppose $c_n = \frac{3n^3+n-100}{2n^2-23n}$. Evaluate $\lim_{n\to\infty} c_n$.

(c) Give an example of a sequence that is monotone but is not convergent.

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