Limit Laws

1 Finite Limits

If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, then limits (including one-sided limits) obey the following laws:

1. Constant Law: $\lim_{x \to a} c = c$
2. I-don't-have-a-name-for-this Law: $\lim_{x \to a} x = a$
3. Sum/Difference Law: $\lim_{x \to a} (f(x) \pm g(x)) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = L \pm M$
4. Product Law: $\lim_{x \to a} (f(x) \cdot g(x)) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = L \cdot M$
5. Quotient Law: $\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M}$, (as long as $M \neq 0$)
6. Power Law: $\lim_{x \to a} f(x)^n = \left(\lim_{x \to a} f(x) \right)^n = L^n$, (if n is a positive integer)
7. Root Law: $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L}$, (for all L if n is odd and for $L \geq 0$ if n is even)

2 Infinite Limits

If one (or both) of $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ are $\pm$, then many limits still obey the above laws, with the following conventions. Let $c > 0$ be a positive real number; then:

- $\infty \pm c = \infty$
- $\infty \cdot c = \infty$
- $\infty \cdot -1 = -\infty$
- $\frac{1}{\infty} = 0$
- $\infty + \infty = \infty$
- $\infty \cdot \infty = \infty$

Indeterminate forms are expressions that cannot be assigned a value. Encountering one of these while solving a limit problem indicates that you should find a different way to evaluate the limit.

$$\frac{0}{0},\ \frac{\infty}{\infty},\ 0 \cdot \infty,\ 1^{\infty},\ \infty - \infty,\ 0^0,\ \infty^0$$

3 Practice

Evaluate the following limits:

a.

$$\lim_{x \to 2} 5$$

b.

$$\lim_{x \to 0^-} \left(x + \frac{1}{x} \right)$$

c.

$$\lim_{x \to 0^+} \left(x \cdot \frac{1}{x} \right)$$

d.

$$\lim_{x \to 0} \left(\frac{x+1}{x-2} \cdot \frac{-1}{x^2} \right)$$

e.

$$\lim_{x \to 0^+} \left(\frac{x}{\ln x} \right)$$