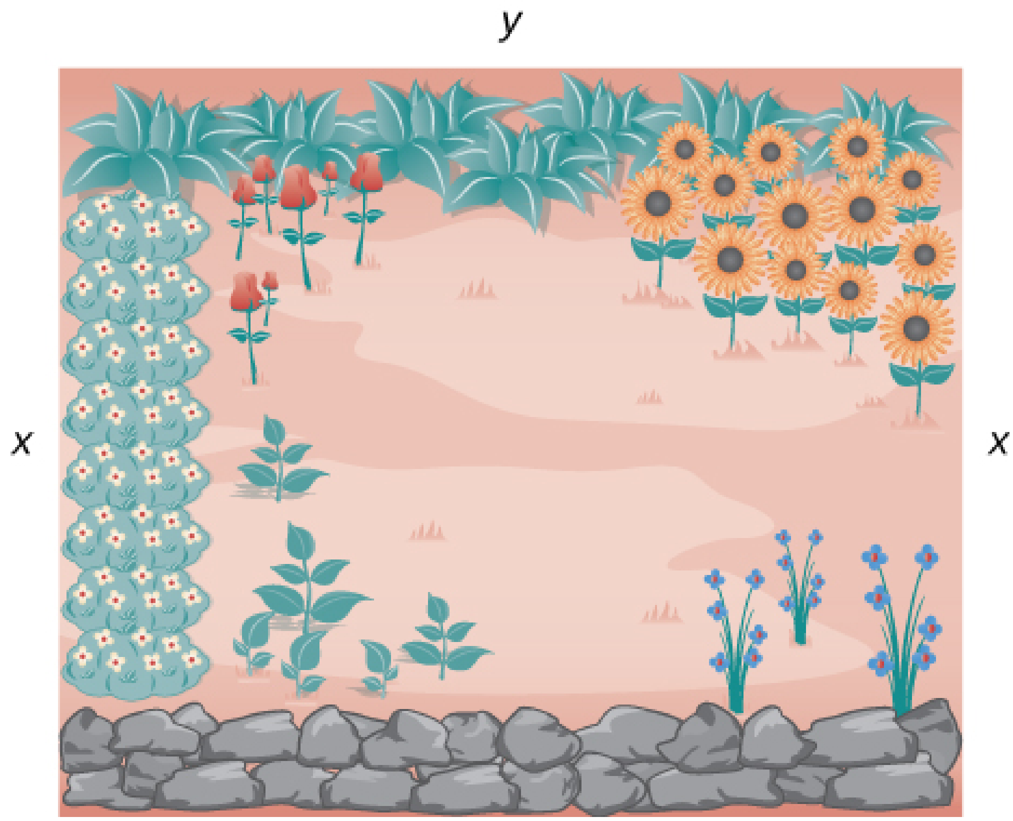
Problem 1. Garden

(a) A rectangular garden is to be constructed using a rock wall as one side of the garden and wire fencing for the other three sides. Given 100 ft of wire fencing, determine the dimensions that would create a garden of maximum area. What is the maximum area?



(b) What would the answer to part (a) be if we had 200 ft of fencing instead of 100 ft?

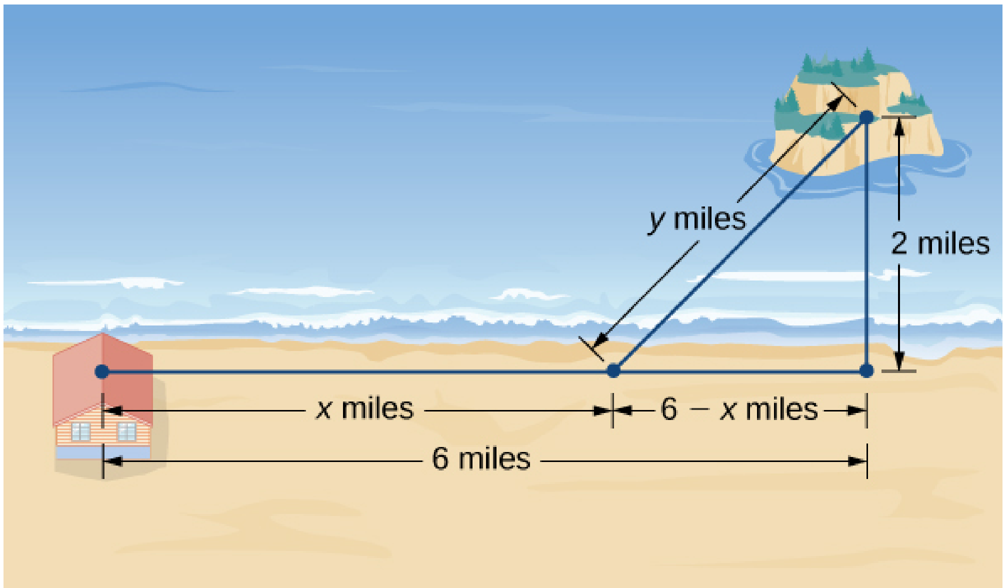
(c) What would the answer to part (a) be if we needed to enclose all four sides in wire fencing, i.e. if the rock wall were gone?

Problem 2. Soup Can

Suppose you are given the task of constructing a soup can that holds 432mL of water (1 mL = 1 cm^3). The material for the can costs \$.02/100cm^2. Find the dimensions of a can that would minimize the cost to produce it.

Problem 3. Island

An island is 2 miles due north of its closest point along a straight shoreline. A visitor is staying at a cabin on the shore that is 6 miles west of that point. The visitor is planning to go from the cabin to the island. Suppose the visitor runs at a rate of 8mph and swims at a rate of 3mph. How far should the visitor run before swimming to minimize the time it takes to reach the island? [Hint: Let x be the distance that the visitor runs along the shore, and let y be the distance that the visitor swims.]



Problem 4. Car Rental

Owners of a car rental company have determined that if they charge customers p dollars per day to rent a car, where $50 \leq p \leq 200$, the number of cars n they rent per day can be modeled by the linear function $n(p) = 1000 - 5p$. If they charge \$50 per day or less, they will rent all their cars. If they charge \$200 per day or more, they will not rent any cars. Assuming the owners plan to charge customers between \$50 per day and \$200 per day to rent a car, how much should they charge to maximize their revenue?

Problem 5. Cargo Ship

A cargo ship is carrying cargo 3 miles offshore and wishes to reach a warehouse located on the shore 5 miles from the current position of the ship. It can deliver the cargo to a transport vehicle that is waiting onshore. The ship can move at a rate of 1 mph and the vehicle can drive onshore at a rate of 3 mph. Where should the ship land in order to reach the warehouse in the least amount of time?

Problem 6. Pizza

A pizzeria can make x pizzas per day. Their cost to make x pizzas is $60 + 3x + \frac{1}{2}x^2$ dollars, and they can sell each pizza for \$15. How many pizzas per day should they make to maximize their profit?

Problem 7. Caffeine

Whenever someone drinks a cup of coffee, the caffeine doesn't enter their bloodstream all at once. It gets absorbed at a rate proportional to some constant α, and simultaneously is eliminated at a rate proportional to some other constant β. We can model the blood concentration of caffeine by the function

$$C(t) = \frac{D}{1 - \frac{\beta}{\alpha}} \left(e^{-\beta t} - e^{-\alpha t}\right)$$

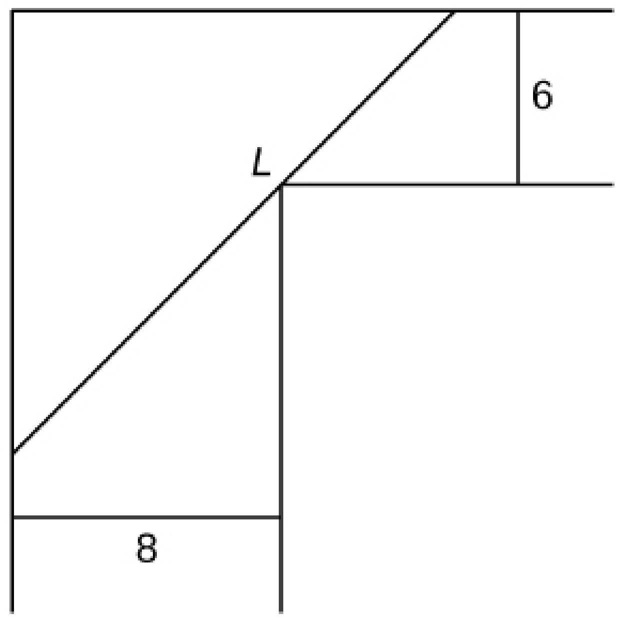
where D is the amount of caffeine consumed.

(a) Find $C'(t)$.

(b) Let's say the average person's body has constants $\alpha = .33$ and $\beta = .21$, respectively. If they drink a cup of coffee (about 70mg of caffeine), how long does it take for their blood concentration of caffeine to reach its maximum value?

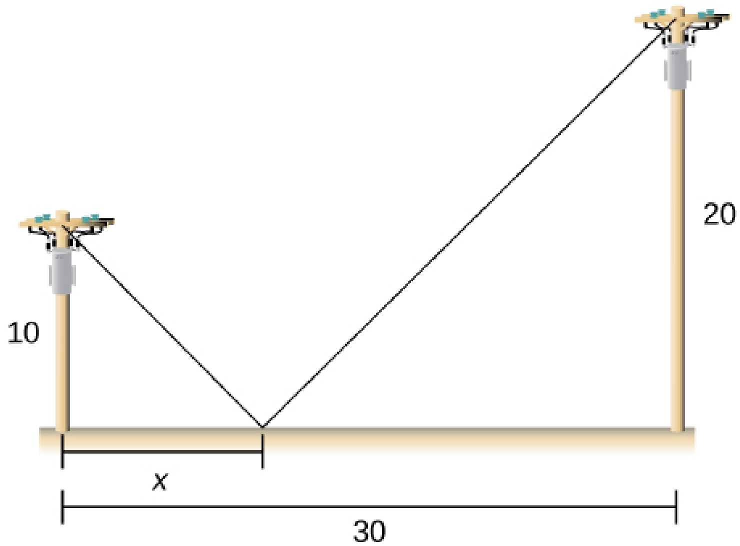
Problem 8. The Moving (1-Dimensional) Sofa Problem

You are moving into a new apartment and notice there is a corner where the hallway narrows from 8 feet to 6 feet. What is the length of the longest item that can be carried horizontally around the corner?



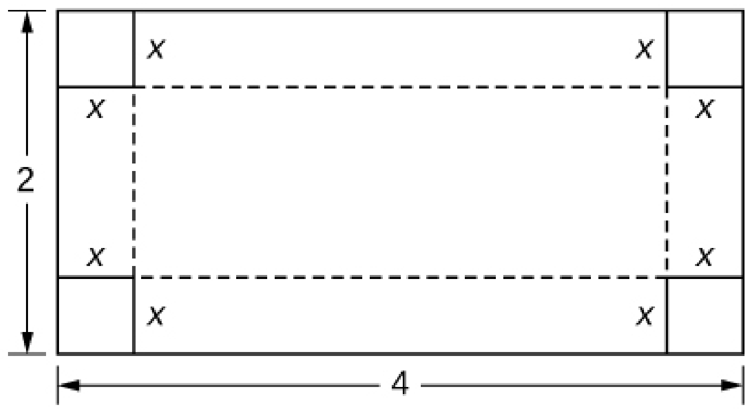
Problem 9. Power Lines

Two poles are connected by a wire that is also connected to the ground. The first pole is 20 feet tall and the second pole is 10 feet tall. There is a distance of 30 feet between the two poles. Where should the wire be anchored to the ground to minimize the amount of wire needed?



Problem 10. Cardboard Box

You are constructing a cardboard box (with an open top) with the dimensions 2 meters by 4 meters. You then cut equal-size squares from each corner so you may fold the edges. What are the dimensions of the box with the largest volume?



Problem 11. Rectangle in an Ellipse

A rectangle is to be inscribed in the ellipse

$$\frac{x^2}{4} + y^2 = 1$$

What should the dimensions of the rectangle be to maximize its area? What is the maximum area?