MATH 1 FALL 2019 : LECTURE 01 MON 09-16-19

SAMUEL TRIPP

Contents

I. Course Information

References

- https://canvas.dartmouth.edu
- https://math.dartmouth.edu/~m1f19

Remarks

Most of the course information can be found at the links provided above, on the paper syllabus handed out today, or by asking me! If you have a question or concern about some aspect of the course, the sooner you get help to resolve it the better.

Assignment!

Sign up on Canvas for a five minute slot to come introduce yourself to me! Available times:
Tuesday, September 17, 11 AM - 12 PM
Tuesday, September 17, 1:15 PM - 2:15 PM
Wednesday, September 18, 3:30 PM - 4:00 PM
...or by appointment.

II. Big Idea Of The Day

Remarks

Functions describe the world! And calculus is the study of how functions change. Today we will learn or refresh the language of sets, and use that language to recall the basics of functions.

III. Functions

Definition

- A **function** consists of a set of inputs, a set of outputs, and a rule for assigning exactly one output to each output. The set of inputs is called the **domain** and the set of outputs is called the **codomain**.
  The notation we use for a function called f with domain the set A and codomain the set B is $f : A \to B$.

The output of a function $f : A \to B$ assigned to the element x in A is denoted $f(x)$.

- The **range** of a function is

$$\{y \text{ in } B \text{ such that there exists } x \text{ in } A \text{ with } f(x) = y\}$$

Example

Are they functions?

Input	Output
0	1
1	4
-1	0
2	9
-2	1

Input	Output
0	1
1	2.3
-1	9
2	9
-2	9

Input	Output
45	93
45	72
60	88
90	91
90	82

Answers: yes, yes, and no. The last example is not contrived: it could take as input the number of minutes studied for the exam, and as output the grade received. But it isn't a function, because not everyone that studies the same amount receives the same grade.

IV. Sets and Notation

Remarks

If we want to get anything done, we need to agree on notation. Let's spend a few minutes doing that.

Content

We will write **sets** of numbers using curly braces, like $\{1, 3, 5, 7, 9\}$ or $\{1, 2, 3, 4\}$ for example. Some sets are so important they get their own names! These include:

- the empty set, $\emptyset$,
- the integers, $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$, denoted $\mathbb{Z}$,
- the natural numbers, $\{0, 1, 2, 3, 4, \ldots\}$, denoted $\mathbb{N}$,
- the rational numbers, numbers of the form $\dfrac{a}{b}$ where a and b are integers, denoted $\mathbb{Q}$, and
- the real numbers, which is any (possibly infinite) decimal expansion, including 7, $\sqrt{2}$, π, and $\dfrac{132}{7}$ for example.

Remarks

Three important pieces of notation to relate sets:

- $\in$ means "is in" or "is an element of"
- $\subseteq$ means "is a subset of"
- $|$ and $:$ mean "such that" and are used to define subsets

Content

Other than just listing elements between curly braces, we also use **set-builder notation**, which is used to define a set by all the numbers that satisfy some property. It is written

$$\{x \in \mathbb{R} \mid x \text{ satisfies some property}\}$$

Some examples are $\mathbb{Q} = \{x \in \mathbb{R} | x = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$ and $\{x \in \mathbb{R} | 1 < x \leq \pi\}$.
How else have you seen the second example written?
Interval notation is used to define subsets of the real numbers: $(a, b) = \{x \in \mathbb{R} | a < x < b\}$ and $[a, b] = \{x \in \mathbb{R} | a \leq x \leq b\}$, and we can mix these up to get $[a, b)$ or $(a, b]$. For sets like $\{x \in \mathbb{R} | x \geq a\}$ we use an ∞ symbol, e.g. $[a, \infty)$.

V. Return to Functions

Practice Problems

Let's find the domain and range of these functions:

- $f(x) = (x-1)^2 - 2$
- $g(x) = \sqrt{1+x}$
- $h(x) = \dfrac{1}{x+3}$

Answers:

- The domain of f is $\mathbb{R}$, and the range is $\mathbb{R}$.
- The domain of g is $[-1, \infty)$ and the range is $[0, \infty)$.
- The domain of h is $\{x \in \mathbb{R} | x \neq -3\}$ and the range is $\{x \in \mathbb{R} | x \neq 0\}$.

Content

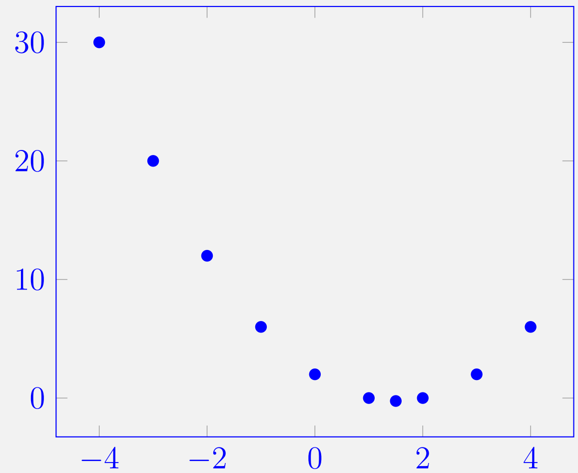
We have seen two ways to present functions so far: tables of inputs and their corresponding outputs, and algebraic formulations as just above. We can also use words to describe the function, such as a function whose input is a person in this classroom, and whose output is their height rounded to the nearest inch, or the function $f(n) =$ the number of prime divisors of n.
The other way to present functions is by their graphs. Given a function f, we can make a picture with the domain on the x-axis and the range on the y-axis, which can help us understand the function.

Practice Problems

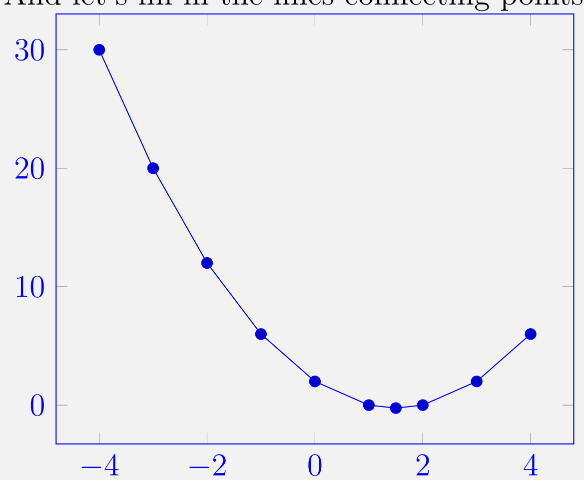
Graph the function $f(x) = x^2 - 3x + 2$.
Answer: first find a few points, then graph those:

x	0	1	-1	2	-2	3	-3	4	-4	3/2
f(x)	2	0	6	0	12	2	20	6	30	-1/4

As a graph:



And let's fill in the lines connecting points:



Remarks

We can tell if a graph is a function by the **vertical line test**. If there is a vertical line that intersects the function more than once, the graph is not a function. Otherwise it is.

Remarks

We go back and forth between these ways of representing functions all the time, taking an algebraic formulation and drawing the graph, by first starting with a table of reference points. Starting with a table of points, or a graph of those, can we come up with a function that fits those? This is the summary of many statistical projects: financial, meteorological, economic, psychological, etc.

Content

We need a few more pieces of notation for functions:

- For a function $f : \mathbb{R} \to \mathbb{R}$, $x \in \mathbb{R}$ is a **zero** of f if $f(x) = 0$.

These are also called **x-intercepts** or **roots**, and are places where the graph crosses the x-axis.

- For a function $f : \mathbb{R} \to \mathbb{R}$, the **y-intercept** is the place where the graph crosses the y-axis, the point $(0, f(0))$.

  Note: the notation for points and open intervals is the same! Usually it is clear what is meant by context.
- A function $f : \mathbb{R} \to \mathbb{R}$ is **increasing** on the interval I if $f(x_1) \leq f(x_2)$ for all x_1 and x_2 in I, with $x_1 \leq x_2$. A function is called **strictly increasing** if all the "less than or equal" signs are replaced by "less than" signs in the above definition. **Decreasing** and **strictly decreasing** are defined analogously. These match up with what we picture in the graphs: strictly increasing means it goes up, and strictly decreasing means it goes down.

Remarks

- How many zeros can a function have? How many y-intercepts can a function have?
- Can a function be increasing and decreasing?

Practice Problems

Find the zeros and y-intercepts of the following functions:

- $f(x) = x^3$
- $g(x) = 2x - 9$
- $h(x) = 2^x$
- $j(x) = x^2 - 4$

Answers:

- 0 is the only zero of f, and 0 is also the y-intercept.
- $\frac{9}{2}$ is the only zero of g, and -9 is the y-intercept.
- There are no zeros of h, and 1 is the y-intercept.
- 2 and -2 are the zeros of j, and -4 is the y-intercept.

For what intervals are the above functions (strictly) increasing and decreasing?

VI. Homework

Assignment!

In addition to the daily homework problems which can be found on the website and the Canvas page, there is an entry survey on the Canvas page which needs to be filled out by Friday at 4 PM.