# MATH 1 FALL 2019 : FIRST X-HR TUE 09-17-19

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## Contents



## I. Big Idea of the Day

**Remarks**

Functions describe the world! Let's talk about functions. A **BIG** idea in mathematics is that if you want to understand some topic, you just need to understand the building blocks, and how to combine these building blocks. Today we will remember some algebra concerning one of the building blocks of all functions, the polynomials.

## II. Polynomials

**Definition**

Polynomials are a pretty general class of functions that come up all the time in math. A **polynomial function** is any function that can be written as

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$$

for some integer $n \geq 0$, and constants $a_n, a_{n-1}, \ldots, a_0$, where $a_n \neq 0$.

**Definition**

The value $n$ is called the **degree** of the polynomial.

- Degree 0 polynomials are called **constant functions**.
- Degree 1 polynomials are called **linear functions**.
- Degree 2 polynomials are called **quadratic functions**.
- Degree 3 polynomials are called **cubic functions**.

A **power function** is any function of the form $f(x) = ax^b$ for real numbers $a$ and $b$. This is just a polynomial with only one term.

## III. Linear Functions

### Remarks

The linear functions mentioned above are very important in mathematics. They are of the form $f(x) = ax + b$ for some constants $a, b \in \mathbb{R}$. Their graphs look like lines; are lines and linear functions basically the same thing?

### Content

Linear functions are some of the most basic functions we see, and are of the form $f(x) = ax + b$, where $a$ and $b$ are real numbers. The two defining features of a line are its slope and where it is, which we can pin down by where it crosses the $y$-axis, called the $y$-intercept.

#### Definition

Consider a line $L$ that passes through two points, which we will call $(x_1, y_1)$ and $(x_2, y_2)$. Then the **slope** of the line, denoted $m$, is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

The slope measures both the steepness and direction of a line.

If we have the line $L$ in the form above, $y = ax + b$, how does the slope relate to $a$ and $b$? Well two points on the line are $(0, b)$ and $(1, a + b)$, so by the above we have that

$$m = \frac{a + b - b}{1 - 0} = a.$$

Similarly, if we plug in 0 for $x$, we see that the $y$-intercept is $(0, b)$.

#### Definition

This is why the above form, $y = mx + b$, is called the **slope-intercept form** of a linear function. But we can write linear functions in other forms too.

- If we know one point on the line, $(x_1, y_1)$, and the slope, $m$, we can write the **point-slope equation** for the line, $y - y_1 = m(x - x_1)$.
- Neither of the last two forms allow us to write vertical lines, which have infinite slope. For that we need the **standard form of a line**, given by $ax + by = c$ for some constants $a, b, c$, with not both $a$ and $b$ zero.

### Practice Problems

Does the line passing through the points $(1, 2)$ and $(3, -7)$ correspond to a linear function? If yes, find the slope-intercept and point-slope form of the line passing through these two points.

## IV. Zeros of Functions

### Content

We find the zeros of a function $f(x)$ by solving the equation $f(x) = 0$. This is easy for degree 0 and degree 1 polynomials, just by algebraic manipulation. For degree 2, we can do this for $f(x) = ax^2 + bx + c$ by using the quadratic formula, which tells us the zeros are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The term $b^2 - 4ac$ is called the **discriminant** and controls the number of solutions; if it is positive, we have two real solutions, if it is 0, we have one real solution, and if it is negative, we have no real solutions.

### Remarks

How do we find the zeros of higher degree polynomials? Is there a cubic or quartic or quintic formula?

## V. Behavior At Infinity

### Content

Sometimes we want to know what happens to the output of a function $f$ as the inputs approach infinity. All sorts of things can happen, and we will study this more (and more rigorously) later in the term.

- Sometimes the value of a function can head towards some finite number as the input grows larger and larger. Can you come up with an example?

  When this happens, if we call the finite number $k$, we say that $f(x)$ approaches $k$ as $x$ approaches infinity, or write $\lim\limits_{x \to \infty} f(x) = k$ or $f(x) \to k$ as $x \to \infty$.
- Sometimes the value of a function grows larger and larger as $x$ grows larger. Can you come up with an example?

  When this happens, we say that $f(x)$ approaches infinity as $x$ approaches infinity, or write $\lim\limits_{x \to \infty} f(x) = \infty$ or $f(x) \to \infty$ as $x \to \infty$.

For polynomials, the coefficient of the leading term determines the function's behavior at infinity. What happens with even degree polynomials as $x \to \infty$? What about $x \to -\infty$? What about odd degree polynomials?