MATH 1 FALL 2019 : LECTURE 03 FRI 09-20-19

SAMUEL TRIPP

Contents

I. Big Idea Of The Day

Remarks

Functions describe the world! Last class we learned about how to combine functions using a number of different operations. But we need some more basic functions to combine. Today we will learn about algebraic and transcendental functions, and see some examples of algebraic functions. Next week we will see some examples of transcendental functions.

II. Algebraic Functions

Remarks

Polynomials are really nice. There are a bunch of other functions that look kind of like polynomials, and are still pretty nice.

Definition

A **power function** is any function of the form $f(x) = ax^b$ for some constants $a, b \in \mathbb{R}$. A **root function** is a function of the form $f(x) = x^{1/n}$ for some integer n greater than 1. Every root function is a power function!

Definition

A **rational function** is a quotient of polynomials.

Content

Combining these basic functions in the ways we have described allows us to generate any **algebraic function**, which is one that involves adding, subtracting, multiplying, dividing, rational powers, and roots.

Most functions that involve various powers of x combined in wacky ways are algebraic functions: $f(x) = \sqrt{\dfrac{x+2}{x^2+2x-9}}$, $g(x) = 2.3x - 4.9$, and $h(x) = 5x + (x^2)^{-\frac{2}{7}}$.

III. Transcendental Functions

Content

Functions that cannot be described by basic algebra are called **transcendental functions**. The most common of these functions are:

- **trigonometric functions**. These relate the lengths of sides of triangles (or "trigons"), and are $\sin(x)$, $\cos(x)$, $\tan(x)$, and their reciprocals.
- **exponential functions**. These are functions of the form $f(x) = b^x$ for some positive real number b other than 1.
- **logarithmic functions**. These are functions of the form $f(x) = \log_b(x)$, where $\log_b(x) = y$ means $b^y = x$, for some positive real number b other than 1.

We are going to come back to all of these and learn what they are in the next week or two!

IV. End Behavior Of Polynomials and Rational Functions

Remarks

Remember from earlier our discussion of "behavior at infinity", which is also called **end behavior** of a function, and is what happens to the function when we plug in bigger and bigger numbers. What happens to $f(x) = x^n$ as we plug in bigger and bigger values of x, for different n?

Content

We can extend this to polynomials. The end behavior of polynomials is controlled entirely by their degree and their leading coefficient.

What can we say about rational functions? If we have $f(x) = \dfrac{g(x)}{h(x)}$ for two polynomials $g(x)$ and $h(x)$, what does the end behavior of f depend on?

Remarks

There is some more notation we skipped last time. We learned before that if a function goes to infinity as x goes to infinity, we can write $f(x) \to \infty$ as $x \to \infty$. Alternatively, we can write $\lim_{x\to\infty} f(x) = \infty$, which means the same thing. If some function goes to a finite number k as x goes to infinity, we would write $\lim_{x\to\infty} f(x) = k$.

We can also ask what happens to $f(x)$ as x gets closer and closer to some other number. Sometimes it goes to a finite number (e.g. $\lim_{x\to 0} x^2+1 = 1$), sometimes it goes to infinity (e.g. $\lim_{x\to 0} 1/x = \infty$), and sometimes it doesn't get closer and closer to anything, but instead jumps around.

V. Domain and Range Refresher

Content

Finding the domain for a function $f(x)$ is checking what values we are allowed to plug in for x. Usually it is easier to figure out what values of x we *can't* plug in! The three most common reasons we can't plug in a value for x:

- we can't divide by zero!
- we can't take the square root of negative numbers!
- sometimes functions just aren't described! (mostly piecewise functions)

Content

Finding the range can be much harder. These are the three basic ways to find the range, and we often use them all to accurately assess the range, because there isn't a straightforward algorithm for finding the range.

- graph! This is always helpful, and you should always do this.
- **decompose your function into smaller pieces, find the range of each, and combine.**
- algebraically.

VI. Transformation of Functions

Remarks

Our last way of combining functions, function composition, is also how we transform functions!

Content

The important part to remember here is that when we graph functions, we put the input on the x-axis and the output on the y-axis. If we want to transform the graph in the x-direction, we need to modify the input. If we want to transform the graph in the y-direction, we need to modify the output.
Modifying the input and output are actually just composing with a function before or after your actual function. Let's do an example. Consider $f(x) = x^2$. What would it mean to shift the graph of this function up by one, or left by one?