MATH 1 FALL 2019 : LECTURE 04 MON 09-23-19

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Contents

I. Big Idea Of The Day

Remarks

Math is useful in the real world. Functions describe the world! Looking at how the rate of change of functions comes up all the time in the world, so let's explore that for a while, then look at mathematical models, or how functions can approximate the real world.

Content

Let's consider the following timetable from the Dartmouth Coach:

Stop	**Time**	**Distance from Hanover**
Hanover	12:00 PM	0mi
Lebanon	12:25 PM	5.7mi
New London	12:55 PM	30.5mi
Boston South Station	2:45 PM	130.5mi
Logan Airport	3:00 PM	134.1

What was the average velocity of the Coach from New London to South Station? What was the average velocity of the Coach from Hanover to Logan Airport?

$$\text{average velocity} = \frac{\text{change in position}}{\text{change in time}}$$

Definition

The **average rate of change** of a function f on the interval $[a, b]$, is

$$\frac{f(b) - f(a)}{b - a}.$$

Content

For linear functions, we can measure their fate of change with a single number! This is the slope. For more complicated functions, the rate of change depends on what part of the function you are at, so is a function itself. We can approximate the rate of change at a point $(a, f(a))$ by taking another point close to a and finding the rate of change of f on $[a, b]$.

Definition

The **secant** to the function $f(x)$ through the points $(a, f(a))$ and $(x, f(x))$ is the line passing through these points, and its slope is given by

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a}$$

Content

As x gets closer and closer to a, the secant becomes a better and better approximation of the rate of change of f at a. The secant lines approach a line that is called the tangent line to f at a, and which measures the rate of change of f at a.

Remarks

There is very common notation we use for this, that you will see over and over again as we go forward. If we want to find the rate of change of f at x, we can approximate it by finding the secant line between $(x, f(x))$, and $(x + h, f(x + h))$ for some very small h. By plugging in to our equation for the slope of the secant line, we can see that the slope of this secant line is

$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{x + h - x} = \frac{f(x+h) - f(x)}{h}$$

II. Mathematical Models

Definition

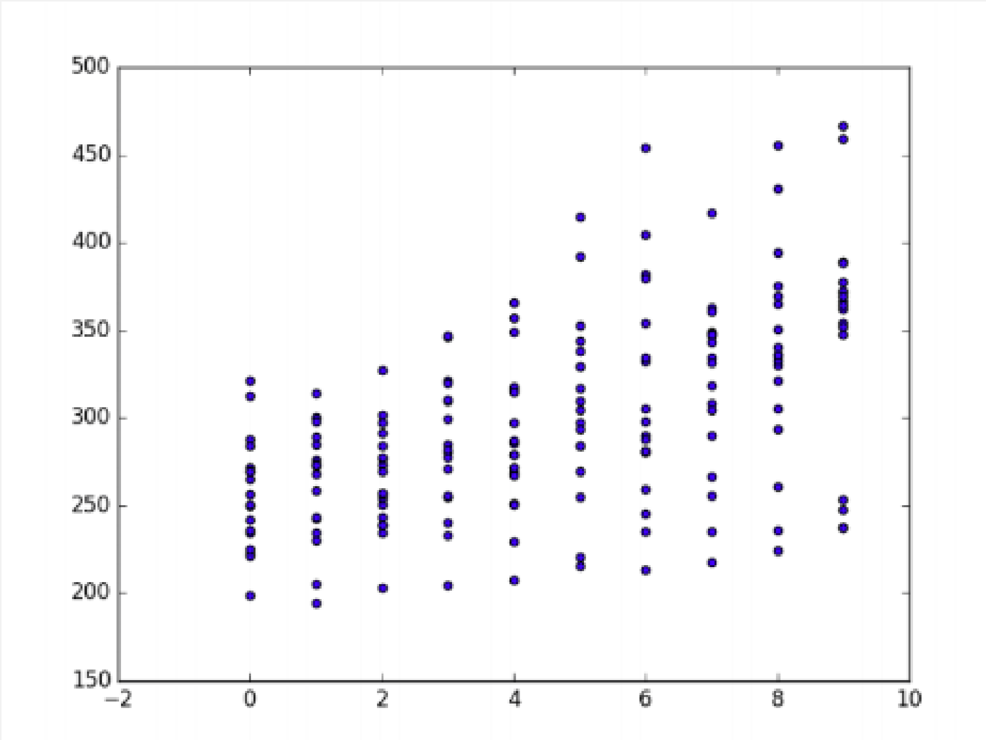
A **mathematical model** is an attempt at taking a real-world phenomenon and representing it physically.

Example

Can we model the location of the Dartmouth Coach with a linear function? If the function takes in the time and outputs the distance, what should be the y-intercept? What should be the slope? Is this a good model?

Example

Reaction Time vs. Days of Sleep Deprivation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