MATH 1 FALL 2019 : LECTURE 05 WED 09-25-19

SAMUEL TRIPP

Contents

I. Big Idea Of The Day

Remarks

Functions describe the world! And we can transform functions by composing them. We can see the effect of these transformations and compositions graphically, which is very helpful.

II. Transformations of functions

Remarks

We understand what it means to move a graph around. We want to translate that into algebra, so we can do it with equations for functions.

Example

What does the function $f(x) = |x|$ look like? What about the function $h(x) = |x| + 5$? How can we write $h(x)$ as the composition of functions, using $f(x)$?

Content

To shift the graph of a function up by c units, just add c to the output of the function, i.e. the graph of $f(x) + c$ looks just like that of $f(x)$ but shifted up by five units. If c is negative, this will shift the function down.

Example

What does the graph of $j(x) = x^2$ look like? What about the function $k(x) = x^2 + 4x + 4$? Why does k look just like j but shifted to the left two? Well, $k(x) = (x + 2)^2$.

Content

To shift the graph of a function right by c units, just subtract c from the input to the function. That is, the graph of $f(x-c)$ is the same as the graph of $f(x)$, just shifted right by c units.

Content

This is all just composing functions! If we want to move the function $f(x)$ in the x-direction, then we need to translate the input before putting it into f. If we want to change f in the y-direction, we need to translate the output. We can define a function $t_c(x) = x + c$, then do $(f \circ t_{-c})(x) = f(t_{-c}(x)) = f(x-c)$ shifts the graph to the right c units. Similarly, $(t_c \circ f)(x) = t_c(f(x)) = f(x) + c$ shifts the graph up c units.
The "t" stands for translate.

Content

In addition to translating our functions, we can scale our functions by some positive constant c. We scale the output by multiplying by c, so $cf(x)$. We scale the input by dividing by c, so $f(x/c)$. This is analogous to how in translation, moving up required adding c but moving right required subtracting, except it is multiplication and division here. If $c > 1$, this is called stretching, and if $c < 1$ then this is called compressing.

Content

If $c = -1$, it doesn't actually stretch our graph, but instead reflects it. We have already seen these reflections! If we multiply our input by $c = -1$, we reflect across the y-axis, and if we multiply our output by $c = -1$, we reflect across the x-axis.

Content

Summary!

Transformation	Effect
$f(x)+c$	Vertical shift by c
$f(x-c)$	Horizontal shift by c
$cf(x)$	Vertical scaling by c
$f(x/c)$	Horizontal scaling by c
$-f(x)$	Reflection across x-axis
$f(-x)$	Reflection across y-axis

III. Trigonometric Functions

Remarks

The trigonometric functions are some of the most important functions in the world! They come up very naturally in the measure of triangles and circles, which, as you know, are everywhere. They are very related to angles, so we need to learn a new way of measuring angles.

Remarks

Historically, you have probably measured angles in a unit called "degrees". There is a somewhat more natural measure of angles called "radian". Just like Celsius and Farenheit, these are different units to measure the same exact thing - angles.

Content

Consider a unit circle: a circle with radius 1. Given an angle θ, the arc it bounds has length s. The radian measure of θ is s; that is the angle that bounds an arc of length 1 has radian measure 1.