Week 1 - Functions

Q: What is a function?

A: a relation in which each input has exactly one output.

Q: Okay, but what is a relation?

A: a set X (the domain), a set Y (the range), and a set of ordered pairs (x, y), where x is in X and y is in Y

Idea

Domain = inputs
Range = outputs

We might draw tables to illustrate relations, e.g.

x	y
0	0
1	1
2	4
3	9

x	y
-5	0
3	0
2	0
$1/2$	0

x	y
-1	-1
0	0
0	1
1	2

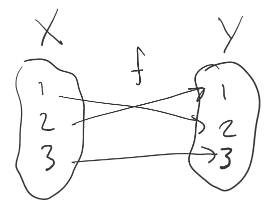
Q: Which of these relations are functions?

Notation

If f is a function and x is an input, then we write $f(x)$, "f of x", when talking about the unique output

Picture "black box" view of functions

$$x \longrightarrow \boxed{f} \longrightarrow y, \text{ a.k.a. } f(x).$$



$\boxed{\text{Warning}}$ Don't get too attached to the letters f, x, and y.

I could have a function called g, that takes inputs called z, etc.

We will write sets of numbers with curly braces like $\{-2, 0, 3\}$.

If x is in a set X, we write $x \in X$. e.g. $3 \in \{1, 2, 3\}$

Some important sets have special symbols, like $\emptyset$, the empty set

$\mathbb{N}$: the set of natural numbers $\{0, 1, 2, 3, \ldots\}$

$\mathbb{Z}$: the set of integers $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{Q}$: the set of rational numbers $\{$fractions p/q for $p, q \in \mathbb{Z}\}$

$\mathbb{R}$: the set of all real numbers (i.e. possibly infinite decimals)

e.g. $0 \in \mathbb{R}$, $-1/2 \in \mathbb{R}$, $\pi \in \mathbb{R}$

$\mathbb{R}$: the set of all real numbers (i.e. possibly infinite decimals) e.g. $0 \in \mathbb{R}$, $-\frac{1}{2} \in \mathbb{R}$, $\pi \in \mathbb{R}$

We use set-builder notation to write sets succinctly:

$\{x \in \mathbb{R} \mid \text{some condition on } x\}$ (some people use : instead of $\mid$)

e.g. $\{x \in \mathbb{R} \mid -1 < x < 2\}$ means "the set of real numbers greater than -1 and less than 2"

Q. How else have you seen this written?

A. $\{x \in \mathbb{R} \mid -1 < x < 2\} = (-1, 2)$

(a, b) means $\{x \in \mathbb{R} \mid a < x < b\}$, $[a, b]$ means $\{x \in \mathbb{R} \mid a \leq x \leq b\}$

and you can also mix the symbols up, like $(a, b]$, $[a, b)$.

Warning: The ordered pair (a, b) can be confused with the open interval (a, b).

For sets like $\{x \in \mathbb{R} \mid x \geq 0\}$, we use an ∞ symbol, e.g. $[0, \infty)$.

Practice

- Find the domain and range of

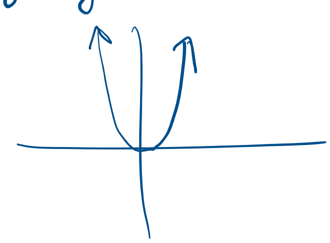
$f(x) = (x+1)^2 + 3$ Domain: $\mathbb{R}$, Range: $[3, \infty)$

$g(x) = \frac{1}{x}$ Domain: $\{x \in \mathbb{R} \mid x \neq 0\}$, Range: $\{x \in \mathbb{R} \mid x \neq 0\}$

Graphing → Geogebra "Transformational View"

Another way of visualizing functions is to graph them.

e.g. $y = f(x) = x^2$



x	x^2
0	0
1	1
-1	1
2	4

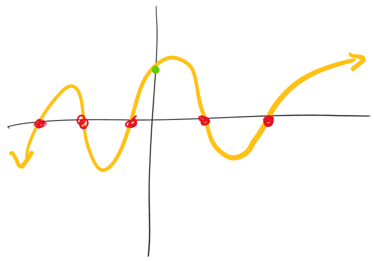
This is hard to do without some reference points, so it is helpful to make a table of values first.

Alternatively, given a graph, you can easily check if it is a function with the vertical line test.

If any vertical line intersects the graph at two or more points, the graph is not of a function.

-1	1
2	4
-2	4

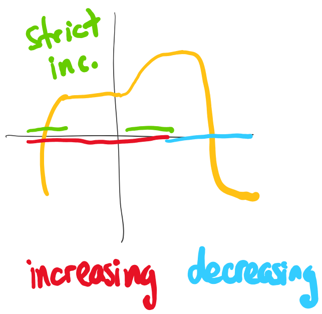
or more points, the graph is <u>not</u> of a function.



Terminology:

Zeroes = x-intercepts = points where the graph crosses the x-axis $\to (x,0)$ where $f(x)=0$

y-intercepts = points where the graph crosses the y-axis $\to (0,f(0))$



We say that f is increasing on the interval I if $f(x_1) \le f(x_2)$ for all $x_1 \le x_2$ in I

We say that f is decreasing on the interval I if $f(x_1) \ge f(x_2)$ for all $x_1 \ge x_2$ in I

Note: Replace $\le$ with $<$ and $\ge$ with $>$ to get definitions of strictly increasing and strictly decreasing

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Practice

Find the zeroes and y-intercepts of the graph $y=f(x)$ for

- $f(x) = 3x+6$ zeroes: $\{(-2,0)\}$, y-int: $(0,6)$
- $f(x) = x^2-3x+2$ zeroes: $\{(1,0),(2,0)\}$, y-int: $(0,2)$
- $f(x) = 2^x$ zeroes: $\emptyset$ y-int: $(0,1)$

For which intervals are the above graphs (strictly) increasing or decreasing?